Abstract: This paper presents the effect of soil-structure interaction (SSI) on seismic inelastic displacement ratios of SDOF systems. Existing methods used in the past assumes the soil is rigid. A simplified equivalent fixed-base method is proposed herein to achieve more accurate estimations for the inelastic displacement demand of a structure. A well-defined degrading model was used to conduct the dynamic analyses. A total of 300 earthquake motions recorded on firm sites, including recent ones from Japan and New Zealand, with magnitudes greater than 5 and peak ground acceleration (PGA) values greater than 0.08g, were selected and scaled to the same hazard level. These earthquake records were applied on five reinforced concrete (RC) columns that were chosen among 255 tested columns based on their beam-column element parameters reported by the Pacific Earthquake Engineering Research Centre. A total of 384,000 dynamic analyses were conducted to derive the required inelastic ratios. Different strength reduction factors and foundation aspect ratios h/r values were assumed for a range of NEHRP soil types C and D properties in the study. The results show that inelastic displacements are relatively greater for slender columns, particularly for high foundation aspect ratios. The large collected data was used to derive mathematical expressions for inelastic displacement ratios, suitable for use in performance-based seismic evaluation in a design office. A new rigorous approach based on fuzzy logic techniques is derived to properly account for the large uncertainty present in the system. The performance evaluation of this approach is evaluated using a series of independent data sets. Accurate results were predicted using the new fuzzy logic model.

Introduction

Most common seismic codes used in design practice assume the base of the foundation to be fixed, which does not consider soil-structure-interaction (SSI). This study attempts to include the effect of SSI in performance based seismic design codes using simplified equivalent single degree of freedom (SDOF) systems. For this purpose, inelastic displacement ratios that account for SSI effects are developed. The proposed hysteretic model accounts for degradation effects using energy-based methods. Five reinforced concrete (RC) columns are selected for analysis amongst the tested columns of the Pacific Earthquake Engineering Research Centre PEER (2014). Using the findings mentioned in Haselton et al. (2008) similar beam-column element parameters are chosen for consistency. A database of 300 earthquake records on firm sites selected in Ozkul (2011) is used in this study. The selection process of the data was based on earthquakes with magnitudes greater than 5 and peak ground acceleration (PGA) values ranging from 0.8g-2.73g. Different soil profiles with effective period and damping values differing from the fixed-base case were compared for different strength reduction factors ranging from 1.5 to 8. Using the equivalent fixed-based method the effects of soil-structure interaction for the SDOF systems are investigated in this paper.

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Review of Previous Work

Performance based seismic design methods are used throughout the world for controlling the earthquake damage. For the case of structural and non-structural elements influenced by lateral deformation, it is important to evaluate the lateral inelastic displacement of a structure accurately; for this purpose the inelastic displacement ratio is estimated in the study for different strength reduction factors. This factor represents the relationship between maximum inelastic displacement and maximum elastic displacement of SDOF systems. Veletsos & Newmark (1960) conducted the first study on this topic in 1960 using three-earthquake ground motions. The results from this study shows the maximum inelastic displacement is approximately equivalent to the maximum elastic displacement for long periods; this is also known as the “equal displacement rule”. Shimazaki & Sozen (1984) identified that the equal displacement rule applies to periods of vibration higher than the “characteristic period”, this is known as the period between short period range and the limiting periods of the response spectra, regardless of which hysteretic model is used. Recently, Miranda (2000) used 264 ground motions to compute the constant ductility inelastic displacement ratios of a SDOF system on firm sites using a series of new simplified functions. He also established that neither earthquake magnitude nor the epicenter distance affects the inelastic displacement ratio. Miranda (2000) also developed a new equation that estimated the inelastic displacement ratios of the SDOF system as a function of ductility ratio and period of vibration. Furthermore, Ruiz-Garcia & Miranda (2003) evaluated existing structures built on firm sites using new simplified functions from a previous study. The inelastic displacement ratios were estimated through evaluating the periods of vibration, level of lateral yielding strength, site conditions, earthquake magnitude, distance to the source and the strain-hardening ratio. This simplified equation became very useful in preliminary design of new structures, however for existing structures the results of the study showed that inelastic displacement ratios underestimate the expected maximum lateral deformations in systems with lateral strength previously known. Similarly, Nassar & Krawinkler (1991) performed analysis on SDOF systems using a smaller set of earthquake records to estimate the inelastic displacement ratios considering the degradation effects with a bilinear, Clough and pinching models. Halabian & El Naggar (2002) mentioned the problems of SSI in the seismic analysis and design of large structures built on soft soils. Chopra & Chintanapakdee (2004) developed two different equations for calculating the inelastic displacement ratios for new and existing structures with 214 earthquake records; however, this model disregarded the degradation effects. The work over recent years shows that Engineer’s interest towards this topic grew with further models, which evaluate the behaviour of soil-structure interaction. However, the effects of SSI for SDOF systems have not been fully investigated yet; particularly the degradation effect of the soil foundation under seismic loading is neglected; it is therefore a challenge in this study to overcome this issue and predict accurate estimations. More recently, Chenouda & Ayoub (2008, 2009) proposed a new energy-based model which accounts for degradation effects to estimate the maximum inelastic displacements of degrading systems. The dynamic analysis also shows predictions of structural collapse for SDOF systems subjected to earthquake loading. Eser et al. (2012) worked on calculating the inelastic displacement ratios of soil interacting systems using effective periods and effective damping values different from the fixed-base. Akyemir (2013) also studied soil interacting systems with the modified Clough model to represent the stiffness degradation of structural systems; an equation for estimating the inelastic deformation of existing structures with lateral strength previously known was proposed as a conclusion. Furthermore, a very recent study by Ozkul et al. (2014) derived expressions for estimating inelastic displacement ratios of degrading fixed-based structures using fuzzy logic techniques. The results of the study confirmed the accuracy of the fuzzy logic model.
Hysteretic Model
A modified degrading Clough model from Clough & Johnston (1966) to represent reinforced concrete columns is used in the analytical simulation. The parameters of the model are defined by three branches; elastic branch, strain-hardening branch and softening branch (cap).

Figure 1. Shows the cycle of one modified Clough model in which the cyclic loading is illustrated with four sections: loading, unloading, reloading and unloading.

Degradation
A method developed in Rahnama & Krawinkler (1993) for modelling the degradation effects is outlined with an eight-parameter energy approach. Four types of cyclic degradation are considered: yield strength, unloading stiffness, accelerated stiffness and cap degradation respectively Chenouda & Ayoub (2008). Degradation is determined based on the following energy-based scalar parameter:

\[ \beta_{str}^i = \left( \frac{E_i}{E_{capacity} - \sum_{j=1}^{i} E_j} \right)^{c_{str}} \]  (1)

\( E_i = \) Hysteretic energy dissipated in the current excursion \( i \);
\( \sum_{j=1}^{i} E_j = \) Total hysteretic energy dissipated in all excursions up to the current one; and
\( E_{capacity} = \) Energy dissipation capacity of the element under consideration;
\( c_{str} = \) Exponent defining the rate of deterioration.

Once the total dissipated energy reaches the same value as the energy dissipation capacity, the columns are then defined as totally degraded (Ozkul, 2011). Referring to the decrease of yield strength, the following expression is then used:

\[ F_y^i = F_y^{i-1} (1 - \beta_{str}^i) \]  (2)

\( F_y^i = \) Yield strength at the current excursion \( i \),
\( F_y^{i-1} = \) Yield strength at the previous excursion \( i - 1 \), and
A similar approach is adopted to account for the other types of degradation.
Soil-Structure Model System

Figure 2. Soil-structure model with soil foundation in Khoshnoudian et al. (2013)

Figure 2. above shows a simple SDOF oscillator resting on a soil foundation; this spring-dashpot-mass model is a concept adopted from Wolf’s (1994) study. The soil-structure model is based on the concept of Cone Models for foundations on the surface of homogenous half-space soil (three-dimensional foundation). The SDOF system is modelled on a foundation layer with a circular disk of radius, \( r \). Soil beneath the foundation is characterised with soil parameters such as shear wave velocity, \( V_s \), dilatational wave velocity, \( V_p \), mass density, \( \rho \), and Poisson’s ratio, \( \nu = 1/3 \). Spring stiffness’s \( (K_x \text{ and } K_\theta) \) and damping coefficients \( (C_x \text{ and } C_\theta) \) are used for the sway and rocking motions of the soil-structure model. To ensure accuracy in the study, relatively accurate soil parameters are gathered from different sources outlined further in the paper.

Table 1 below represents the stiffness and damping coefficient of the horizontal and torsional cones deforming in shear defined as \( (K_x, C_x) \) respectively. In addition, the stiffness and damping coefficient of the vertical and rocking cones deforming axially are defined as \( (K_\theta, C_\theta) \) respectively.

<table>
<thead>
<tr>
<th>Stiffness and damping coefficients of Cone Model Wolf (1994)</th>
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<tbody>
<tr>
<td><strong>Horizontal Stiffness Coefficient:</strong></td>
</tr>
<tr>
<td>( K_x = \frac{8.\rho.\frac{V_s^2}{2-\nu}.r}{2-\nu} )</td>
</tr>
<tr>
<td><strong>Vertical Stiffness Coefficient:</strong></td>
</tr>
<tr>
<td>( K_\theta = \frac{8.\rho.\frac{V_s^2}{3}.r^3}{3.(1-\nu)} )</td>
</tr>
<tr>
<td><strong>Horizontal Damping Coefficient:</strong></td>
</tr>
<tr>
<td>( C_x = \rho.\frac{V_s.\pi.}{3}.r^2 )</td>
</tr>
<tr>
<td><strong>Rocking Damping Coefficient:</strong></td>
</tr>
<tr>
<td>( C_\theta = \rho.\frac{V_p.\pi.}{3}.r^4 )</td>
</tr>
</tbody>
</table>

From this the dilatational wave velocity, \( V_p \) is calculated by rearranging the equations represented in Wolf (1994):

\[
V_p = V_s \sqrt{\frac{1-\nu}{1-2.\nu}}
\]
Equations (3-7) are based on the Truncated Cone Model from Meek & Wolf (1992), used in the equivalent fixed-base model. It is important to account for the damping of the soil-structure system in order to determine the maximum displacement of the SDOF system. To investigate the soil-structure interaction more accurately, the stiffness and damping coefficients are used to derive an effective period and damping of the soil-interacting system.

**Analysis Methodology**

The inelastic displacement ratio of an SDOF system with soil-structure interaction (SSI) is evaluated similar to the fixed-base model in terms of constant yield strength (Aydemir, 2013); this simplifies the analysis without having to conduct any further iterations. The analysis is conducted on SDOF structures with a period range 0.2-1.4s for different values of strength reduction factors ranging. Overall, 300 different earthquake records are used for the fixed and flexible foundation model.

The effect of soil-structure interaction (SSI) is accounted for by considering the effective period, $\tilde{T}$, and effective damping, $\beta$, of the SDOF replacement oscillator system (Veletsos & Newmark, 1960). These values are outlined in FEMA (Federal Emergency Management Agency, 2003). The effective period of the system with soil-structure interaction is:

$$\tilde{T} = T \sqrt{1 + \frac{k}{K_x} \left(1 + \frac{K_x h^2}{K_\theta}\right)} \quad (8)$$

And the effective damping of the system with soil-structure interaction is:

$$\tilde{\beta} = \beta_0 + \frac{0.05}{T^{3/4}} \quad (9)$$

Where the $\beta_0$ denotes the foundation-damping factor as mentioned in the current US codes FEMA (2003).

**Data Collection and Model Validation**

In this study a total of 300 earthquake records were collected from three-ground motion database. The data collection was carried out in Ozkul (2011) and was used for this project to increase statistical significance. These records were selected from the Kyoshin Network (K-Net), GeoNet, and with the majority of the records, 266, from the PEER ground motion database. The earthquake motions were detected to correspond to soil type C and D according to the NEHRP soil classification in FEMA (2003), with peak ground acceleration (PGA) values varying from 0.08g to 2.73g. The $Sa (T_1)$ scaling method is chosen in this study because of its efficiency and simplicity. Five specimens were chosen from the PEER Structural Database to simulate typical deformation behaviour of reinforced concrete columns. The model parameters for these columns are identified in Ozkul (2011). Using the model parameters and generating 40 different periods of vibration ranging from 0.2s to 1.4s, inelastic displacement ratios for fixed-base and SSI systems are established as detailed below.

**Displacement Estimates of RC Columns**

Figure 3. Below shows the inelastic displacement ratios (IDR) of the cases considered for strength reduction factor $q=3$ and 4. Overall, there is a period shift between the fixed-base and soil cases selected, however, the change in effective periods were more evident in cases where the structure is more slender (aspect ratio $h/r = 5$), and for softer soil conditions (soil type D).
As the strength reduction factor increases, the inelastic displacement ratio is likely to increase. In view of soil type D, the collapse at a higher strength reduction factor indicates a larger inelastic displacement response in comparison to soil type C and fixed-base foundation, where the collapse in the system occurred at an early stage in the simulations. The IDR values after 0.6 seconds somewhat follow the equal displacement rule (i.e. maximum inelastic displacement approximately equals to maximum elastic one).

The calculated inelastic displacement for soil type D, and \( h/r = 5 \) results in inelastic deformations greater than those of other soil types and the fixed-base foundation. Although, the IDR value for the fixed-base is higher, the elastic displacement of the system is actually lower. The final inelastic displacement is found to be higher as the soil conditions become softer and as the aspect ratio increases.

These results show a great significance in including SSI effects in earthquake design of structures, particularly for soft soil type D and large aspect ratios, in the case of periods less than 0.6s. However, for periods greater than 0.6s the IDR values approximately equal to one, satisfying the equal displacement rule.

**Fuzzy Logic-Based IDR**

The Fuzzy logic approach was originally introduced by Zadeh (1965) and has been adopted in various engineering problems since. Incorporation of fuzzy logic in earthquake engineering applications was conducted in earlier studies [e.g. Furuta, 1993, Wadia-Fascetti and Gunes, 2000]. The use of a fuzzy logic approach to estimate seismic demands was not performed before. This study aims at adopting a fuzzy logic approach to estimate the inelastic displacement ratios of SDOF structures.

To develop fuzzy logic-based IDR functions for fixed-based conditions, the strength reduction factor (q) and period of vibration (T) were considered as the fundamental fuzzy input variables having uncertain boundaries in this study. The Takagi-Sugeno fuzzy logic approach (1985) was used in this study to develop the IDR functions. The analytical model results obtained in the previous section were used for this new development purpose. Consideration of eight strength reduction factors (q) and 40 periods of vibration (T) resulted in 320 inelastic displacement ratio points in total. These data points were divided into training and testing data. The training (calibration) data consisted of 70% of the data, which were randomly selected, and was used to establish the fuzzy logic model; whereas, the testing
(prediction) data consisted of the remaining 30% of the data, and was used to validate the model.

At the end of the training process, three fuzzy sets were defined for each of the input variable qualifying their uncertainties with linguistic expressions such as low, medium and high. Gaussian membership functions were used to establish those fuzzy sets for \( q \) and \( T \) respectively. Nine fuzzy rules with three fuzzy sets for each input variable were then optimized as depicted in Table 2. In order to evaluate the inelastic displacement ratio (IDR), first the weight of each IDR function was calculated and then the weighted average of the nine IDR functions was evaluated using the following equations:

\[
r_r = (m_T^r \ast m_R^r) \quad \text{and} \quad (10)
\]

\[
\text{IDR} = \frac{\sum_{r=1}^{9} (r_r \ast \text{IDR}_r)}{\sum_{r=1}^{9} (r_r)} \quad (11)
\]

where \( m_T^r \) and \( m_R^r \) represent respectively the degrees of membership of the period and strength reduction factor fuzzy sets of the \( r \)th rule.

<table>
<thead>
<tr>
<th>RULE</th>
<th>DESCRIPTION</th>
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<tbody>
<tr>
<td>( q_1 )</td>
<td>IF ( T ) is Low and ( q ) is Low THEN IDR(_{1}) = -5.39 + 0.1014 ( T ) + 1.898 ( q )</td>
</tr>
<tr>
<td>( q_2 )</td>
<td>IF ( T ) is Low and ( q ) is Medium THEN IDR(_{2}) = -34.39 - 0.1594 ( T ) + 13.26 ( q )</td>
</tr>
<tr>
<td>( q_3 )</td>
<td>IF ( T ) is Low and ( q ) is High THEN IDR(_{3}) = 21.81 - 0.8248 ( T ) + 4.139 ( q )</td>
</tr>
<tr>
<td>( q_4 )</td>
<td>IF ( T ) is Medium and ( q ) is Low THEN IDR(_{4}) = -1.812 + 0.0177 ( T ) + 1.442 ( q )</td>
</tr>
<tr>
<td>( q_5 )</td>
<td>IF ( T ) is Medium and ( q ) is Medium THEN IDR(_{5}) = -3.131 + 0.09987 ( T ) + 1.482 ( q )</td>
</tr>
<tr>
<td>( q_6 )</td>
<td>IF ( T ) is Medium and ( q ) is High THEN IDR(_{6}) = -5.499 + 0.209 ( T ) + 1.141 ( q )</td>
</tr>
<tr>
<td>( q_7 )</td>
<td>IF ( T ) is High and ( q ) is Low THEN IDR(_{7}) = -0.2643 - 0.159 ( T ) + 2.533 ( q )</td>
</tr>
<tr>
<td>( q_8 )</td>
<td>IF ( T ) is High and ( q ) is Medium THEN IDR(_{8}) = -5.557 - 0.1276 ( T ) + 3.765 ( q )</td>
</tr>
<tr>
<td>( q_9 )</td>
<td>IF ( T ) is High and ( q ) is High THEN IDR(_{9}) = -1.049 - 0.189 ( T ) + 6.607 ( q )</td>
</tr>
</tbody>
</table>

After the new model that estimates the IDR was established using the training data, the testing data was used for validating the method. As an example, the graphical representation of fuzzy logic model predictions using testing data for \( q=4 \) is shown Figure 4. It is noteworthy to mention that all of the testing plots result in one conclusion: IDR predictions of the fuzzy logic-based model show remarkably good agreement with the synthetic data.

![Figure 4. Inelastic Displacement Ratio for q=4](image-url)
The performance evaluation of the Fuzzy Logic model is shown in Figure 5, which depicts the relationship between the synthetic data and the data predicted using the proposed model for all $q$ values considered. Most of the predicted values are around a 45° diagonal line, with a Coefficient of Efficiency value equal to 0.91. It can be concluded that the proposed fuzzy logic model is a viable approach that estimates the inelastic displacement ratio of moderately degrading SDOF RC structures with high accuracy, which makes it a good potential alternative to existing design guidelines.

![Fuzzy Logic Method Coef. of Efficiency = 0.91](image)

**Figure 5. Model Data (Synthetic) versus Fuzzy Logic Predicted Data**

Extension of the fuzzy logic approach to account for soil flexibility is currently underway. In this case, the aspect ratio $h/r$ is assumed as a third fuzzy input variable.

**Conclusion**

From the results of this study the following conclusions can be drawn:

1. The effects of soil-structure interaction for soil type C with aspect ratio, $h/r = 1$ & 3 are negligible, whereas for soil type D and $h/r = 5$ it should be considered in the performance-based earthquake design process. In this case, particularly for short period region, the IDR for the interacting system and fixed-base are considerably different.
2. Soil types C & D show lower IDR's than the fixed-base case. The difference of IDR's was clearer for soil type D, in comparison to soil type C. In general, the effect of soil-structure interaction should be considered in this case for strength reduction factors, $q > 3$.
3. From the results, higher IDR values do not represent the most critical case, however, when the maximum inelastic displacement of the system is to be determined. It showed that soil type D was the most critical.
4. The aspect ratio ($h/r$) used in the study is an important parameter to represent the IDR. From the results, there is a decrease in IDR values for $h/r = 5$ in comparison to other cases where $h/r = 1$ & 3 to a period less than 0.6s. For periods longer than that, the IDR values satisfy the equal displacement rule. Therefore, for periods less than 0.6s the effect of SSI should be considered in seismic design for high aspect ratio values.
5. Predicted IDR values obtained using the fuzzy logic model matches the experimental data with great accuracy. The fuzzy logic model can be therefore considered as an excellent approach to estimate IDR values.
REFERENCES


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