

INCORPORATING DURATION EFFECTS IN DESIGN COLLAPSE CAPACITY SPECTRA OF DUCTILE SYSTEMS

Maria LIAPOPOULOU¹, Miguel BRAVO-HARO² & Ahmed Y. ELGHAZOULI³

Abstract: *The seismic collapse capacity of ductile single-degree-of-freedom systems vulnerable to $P-\Delta$ effects is investigated in this paper, with the aim of pointing out the differences due to ground motion duration. The main objective is to incorporate duration effects in collapse capacity assessment procedures of modern ductile systems and provide simple relationships to predict the structural collapse capacity. In order to separate the effect of duration from other characteristics of the earthquake records, 101 pairs of long and short records with equivalent spectral response are assembled. The considered structural systems exhibit a trilinear backbone curve consisting of an elastic, a hardening and a negative stiffness segment, whose slope is related to the level of $P-\Delta$. The parameters under investigation are the fundamental period, the slope of the negative stiffness branch, the ductility and the strain hardening ratio. Regarding the hysteretic behaviour, bilinear and pinching models with no strength degradation are examined. Incremental dynamic analysis is employed to determine the collapse capacities of the considered systems, and design collapse capacity spectra are derived through nonlinear regression analyses. The results show that the collapse capacity reduces with increasing duration and decreasing period and ductility. Pinching hysteretic models are related to higher collapse capacities than their bilinear counterparts, while $P-\Delta$ effects are shown to be detrimental, as expected. Overall, it is recommended that duration effects should be considered in the assessment of collapse capacity, as their disregard could result in non-negligible overestimations, especially in the case of bilinear systems.*

Introduction

Seismic collapse of a structural system is defined herein as the state at which a finite increase of the ground motion intensity would cause infinite increase of displacement demands in the structure (Haselton et al., 2009). This can generally be attributed to $P-\Delta$ effects, degradation of material, or both (Miranda and Akkar, 2003). During the last decades, extensive research has been carried out towards the assessment of seismic collapse capacity of structures, focusing on degrading or non-degrading multi-degree-of-freedom (MDOF) or single-degree-of-freedom (SDOF) systems. The focus of this paper is on non-degrading SDOF systems; hence, only the main studies considering such systems of interest are briefly summarised in the following.

Miranda and Akkar (2003) studied the collapse capacity of bilinear SDOF systems subjected to a suite of 72 ground motion records. They provided collapse capacity spectra as a function of the fundamental period and the slope of the negative stiffness branch and concluded that both lower levels of post-yield stiffness, which is equivalent to lower levels of $P-\Delta$, and higher values of structural period increase the collapse capacity. These findings were confirmed by Adam and Jäger (2012), who used bilinear, peak-oriented and pinching hysteretic models and an assembly of 44 far-field earthquake records to study the collapse of SDOF systems. The authors considered $P-\Delta$ effects through the application of gravity load, which causes rotation of the backbone curve, and developed collapse capacity spectra as a function of the period, the level of $P-\Delta$, the viscous damping and the type of hysteresis. These spectra were later refined by Tsantaki et al. (2015). The more general case of ductile systems with trilinear backbones was examined by Vamvatsikos et al. (2009), through deconstruction of the problem to determining the collapse capacity of a bilinear and an elastic-perfectly plastic system. Based on this simplification, an expression for the collapse capacity for different values of period, negative slope, ductility and strain hardening was derived.

¹ PhD student, Imperial College, London, United Kingdom, maria.liapopoulou17@imperial.ac.uk

² Postdoctoral researcher, Imperial College, London, United Kingdom

³ Professor, Imperial College, London, United Kingdom

However, none of these studies considered the role of duration in the assessment of collapse capacity. Attempts to incorporate duration in this process have only recently been made, all proving its importance. Foschaar *et al.* (2012) compared collapse capacities of a steel concentrically braced frame subjected to the FEMA P695 set (FEMA, 2009) and sets of long duration records, which were assembled based on different duration metrics. They reported a maximum of 60% reduction in collapse capacity of the long duration set based on the 5-95% significant duration compared to the FEMA P695 set. Although the authors pointed out the importance of considering spectral shape for an accurate assessment of duration effects, this was not addressed in their study. Raghunandan and Liel (2013) employed the same definition of duration for the assessment of reinforced concrete frames subjected to long and short duration records and reported up to 56% reduction in median collapse capacity of the latter set. The differences in spectral shape characteristics of the two sets were assumed to be captured by using spectral displacement as the intensity measure. In another study, Raghunandan *et al.* (2015) reported an average reduction of 36% in median collapse capacity of ductile reinforced concrete frames and 12% for non-ductile ones, when subjected to long duration earthquakes. The effect of spectral shape was later explicitly considered by Chandramohan *et al.* (2016), who assembled short and long records with equivalent response spectra to investigate the effect of duration on a steel moment resisting frame and a reinforced concrete bridge pier, recording 29% and 17% reduction in collapse capacity respectively, when long records were used. Bravo-Haro and Elghazouli (2018) assembled 77 short and 77 response spectrally equivalent long records to conduct analyses of 4 moment resisting frames and 50 equivalent single-degree-of-freedom (ESDOF) systems. They found up to 24% reduction in collapse capacity of the considered MDOF systems due to longer duration, which attains 40% for highly degrading ESDOF models.

The objective of the present research is to quantify duration effects in collapse assessment of ductile SDOF systems through conducting a parametric investigation of the importance of duration in conjunction with other properties, including the fundamental period, negative post-capping stiffness, ductility capacity, strain hardening ratio, and hysteretic behaviour. Incremental dynamic analysis (IDA) (Vamvatsikos and Cornell, 2002) and the technique of response spectrally equivalent records (Chandramohan *et al.*, 2016) are employed to determine the collapse capacities of the considered SDOF structures. Based on the results, collapse capacity spectra are developed for non-degrading ductile systems.

Methodology

In this section the details of the SDOF system, ground motion records and analysis method are presented.

SDOF model

An SDOF model with trilinear backbone and varying levels of fundamental period, strain hardening ratio, ductility and negative stiffness is employed. The examined values of each parameter are given in Table 1. Regarding the hysteretic behaviour, bilinear and pinching hysteresis with no strength or stiffness deterioration are considered.

Parameter	Values considered
Period, T (s)	0.2, 0.5, 0.7, 1.0, 2.0, 3.0
Negative stiffness, θ - α	0.02, 0.04, 0.06, 0.10, 0.20, 0.30
Ductility, μ_c	2.0, 4.0, 6.0
Strain hardening ratio, α_h	0.00, 0.02, 0.05, 0.10

Table 1. Parameters defining the SDOF systems.

The SDOF model consists of a pinned rigid element of unit height with a rotational spring and damper at its base. Rayleigh damping equal to 5% is assigned to the damper. The mass, which is tuned accordingly to attain a given fundamental period, is concentrated at the tip of the inverted pendulum. P- Δ effects are considered through the application of gravity loads at the tip, such that the previously specified levels of negative stiffness of the post-capping branch are achieved. The post-capping stiffness before the application of the gravity loads is equal to 3% of the initial elastic stiffness ($\alpha=0.03$). It is noted here that the strain hardening ratio refers to the stiffness of the intermediate branch without P- Δ effects. The considered SDOF system is modelled in OpenSees (McKenna *et al.*, 2011) and is depicted in Figure 1, along with its moment-rotation curve with and without gravity loads.

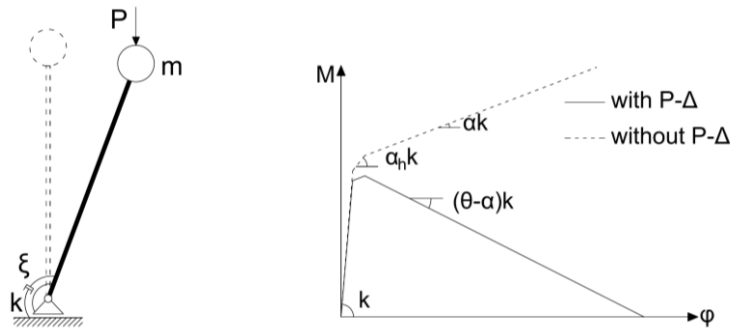


Figure 1. SDOF model and moment-rotation curve.

Earthquake records

In order to study the effect of duration, long and short ground motion records with equivalent spectral response are assembled, such that any differences between them can be explained by their different duration characteristics. Following the method suggested by Chandramohan *et al.* (2016), a database of 101 long and short records is compiled. The 5-95% significant duration is selected to characterise strong motion duration, with a threshold of 25s indicating the transition from short to long earthquakes. The 50th, 16th and 84th percentile acceleration spectra of each set of records are plotted in Figure 2. The seismic events used to assemble short and long duration records are summarised in Tables 2 and 3 respectively.

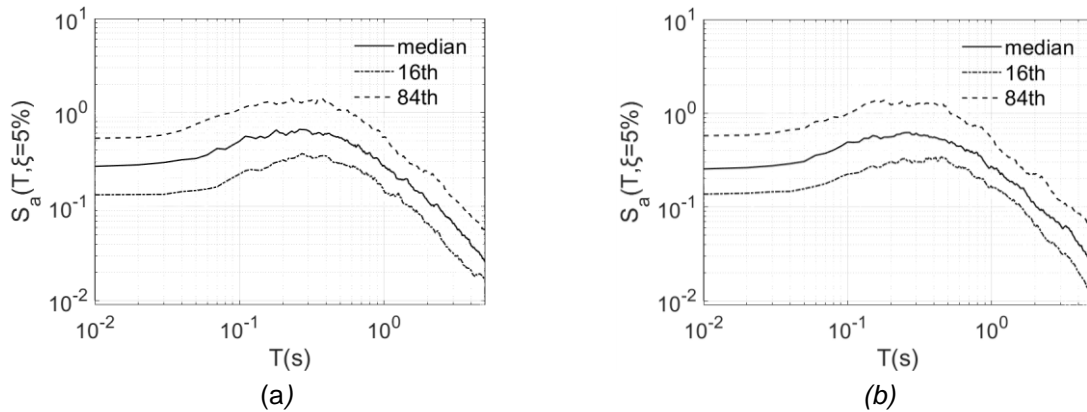


Figure 2. Median, 16th and 84th acceleration response spectra in logarithmic scale of (a) long duration set and (b) short duration set.

Earthquake	Number of records	Earthquake	Number of records
1999 Chi-Chi, Taiwan	24	1981 Taiwan	3
1992 Cape Mendocino	1	2003 Big Bear City	1
1994 Northridge	8	1992 Erzican, Turkey	1
1986 Taiwan	1	2007 Chuetsu-oki	6
1999 Hector Mine	3	1984 Morgan Hill	2
2010 Darfield, New Zealand	3	1980 Irpinia, Italy	3
1989 Loma Prieta	2	2000 Yountville	1
1983 Coalinga	3	1997 Northwest China	1
2004 Niigata, Japan	2	2004 Parkfield	3
1983 Mammoth Lakes	1	1991 Sierra Madre	1
1979 Coyote Lake	2	1979 Imperial Valley	5
1987 Whittier Narrows	6	1995 Kobe, Japan	1
2008 Iwate	2	1992 Landers	4
1999 Kocaeli, Turkey	1	2010 El Mayor-Cucapah	2
2009 L'Aquila, Italy	1	1976 Friuli (aftershock 13), Italy	1
1986 Chalfant Valley	2	1980 Livermore	1
1992 Cape Mendocino	1	2011 Christchurch	2

Table 2. Short duration earthquakes.

Earthquake	Number of records
1985 Mexico City, Mexico	8
1992 Landers	6
1999 Kocaeli, Turkey	4
2003 Hokkaido, Japan	12
2010 El Mayor-Cucapah	6
2010 Maule, Chile	16
2011 Tohoku, Japan	49

Table 3. Long duration earthquakes.

Incremental dynamic analysis

The examined SDOF systems are subjected to the suite of 101 long and short records, which are scaled to increasing levels of intensity up to the state of collapse, as prescribed by IDA (Vamvatsikos and Cornell, 2002). The 5% damped spectral acceleration at the fundamental period is chosen as the intensity measure, while the displacement at the tip of the SDOF model is recorded as the engineering demand parameter. An increase of the recorded displacement above the maximum value that the system can sustain, as determined by its force-deformation curve, or flattening of the IDA curve are both considered to signalise structural collapse. The IDA curves are summarised into their 16%, 50% and 84% fractiles and accordingly, the 16%, 50% and 84% collapse capacities are determined. The focus of this paper is on the median collapse capacity, henceforth referred to as collapse capacity. The analyses are performed in MATLAB and OpenSees (McKenna et al., 2011) using the codes by Vamvatsikos and Cornell (2004), after appropriate modifications for the needs of this research.

Collapse capacity

The role of different structural parameters in the collapse capacity of SDOF systems subjected to short and long ground motion records is assessed in this section. The figures that follow show that long duration seismic events result in a reduction of collapse capacity, while the amount of this reduction depends on the structural system's parameters.

Figure 3 illustrates the effect of period on the collapse capacity of a bilinear and a pinching system with the same characteristics ($\theta-\alpha=0.02$, $\mu_c=6.0$ and $a_h=0.00$). It is evident that in both cases an increase in period causes an increase in collapse capacity, which implies that the general trend in collapse capacity is unaffected by the hysteretic behaviour. Therefore, only pinching hysteresis is considered in the rest of this section. Nevertheless, it is worth noticing that the main difference of the two systems is that higher collapse capacities are observed for pinching hysteresis. With respect to the effect of ground motion duration, it seems to be more significant for longer periods and bilinear hysteresis, as greater reductions of collapse capacity are recorded in this case.

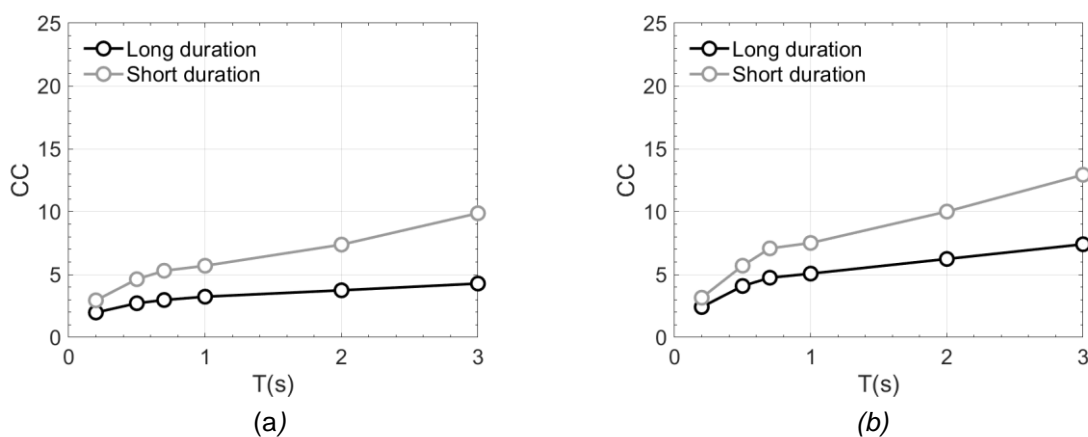


Figure 3. Effect of fundamental period on collapse capacity for a system with $\theta-\alpha=0.02$, $\mu_c=6.0$, $a_h=0.00$ and (a) bilinear hysteresis, (b) pinching hysteresis.

The collapse capacity of a pinching system is plotted versus the level of P- Δ and the strain hardening ratio in Figures 4a and 4b respectively, based on which the detrimental effect of the former and the crucial role of the latter can readily be confirmed. From Figure 4a it becomes clear

that high levels of $P-\Delta$ are associated with extremely low collapse capacities, almost equal to unity, which implies an immediate collapse at the end of the elasticity. In this case, hardly any differences between short and long records can be discerned. On the contrary, the reduction of collapse capacity due to duration becomes increasingly important, as the second order effects reduce. Figure 4b indicates that strain hardening contributes towards higher resistance to collapse, while no strong dependence of duration effects on strain hardening can be perceived.

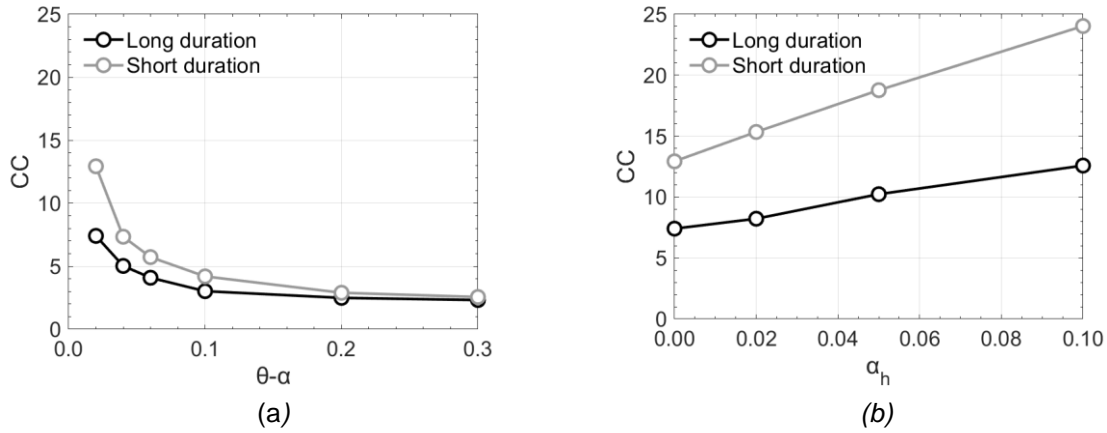


Figure 4. Effect of (a) level of $P-\Delta$ on collapse capacity for a system with $T=3.0s$, $\mu_c=6.0$, $a_h=0.00$ and pinching hysteresis, and (b) strain hardening ratio on collapse capacity for a system with $T=3.0s$, $\theta-\alpha=0.02$, $\mu_c=6.0$ and pinching hysteresis.

When examining the variation of collapse capacity with the ductility, a distinction must be made based on the relative values of strain hardening ratio, α_h , and rotation due to $P-\Delta$ effects, θ . If the former is higher or equal to the latter (i.e. $\alpha_h \geq \theta$), then the stiffness of the intermediate branch of the force-deformation curve after the application of gravity loads remains positive or zero, and hence, it exerts a positive influence on the collapse capacity, increasing the system's resistance to collapse. Therefore, higher ductility capacity, which implies longer intermediate branch, leads to higher collapse capacities, as depicted in Figure 5b. On the contrary, the opposite holds true if the rotation caused by $P-\Delta$ is large enough to cause a negative slope of the intermediate branch (i.e. $\alpha_h < \theta$), thus turning its influence to be adverse, as in Figure 5a, where a decreasing trend of collapse capacity with ductility is discerned. However, in both cases the reduction of collapse capacity due to duration appears not to depend on the value of ductility or strain hardening ratio.

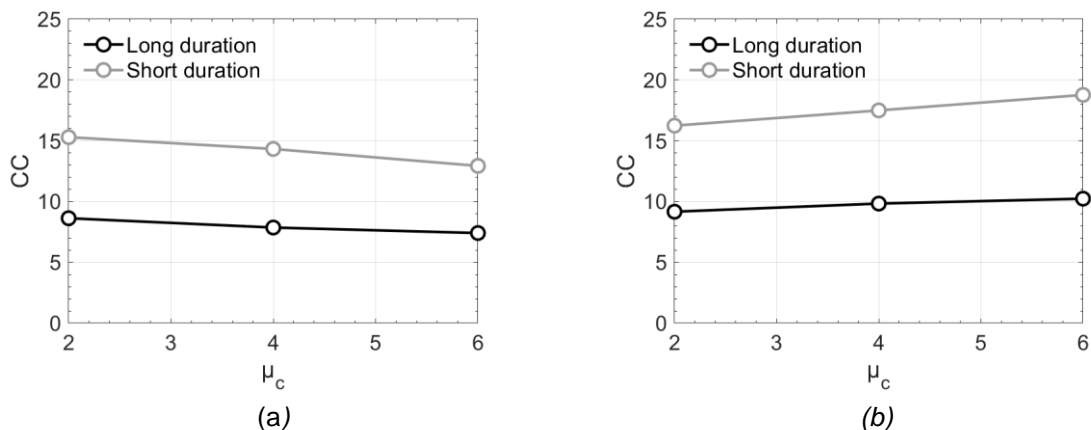


Figure 5. Effect of ductility on collapse capacity for a system with $T=3.0s$, $\theta-\alpha=0.02$, pinching hysteresis and (a) $a_h=0.00$, (b) $a_h=0.05$.

Collapse fragility curves

In this section collapse fragility curves are plotted for short and long duration records. Fitted collapse fragility curves to the data obtained by the analyses are presented, which are computed based on the assumption of lognormal distribution of the collapse capacity.

A reference system with $T=3.0s$, $\theta-\alpha=0.02$, $\mu_c=6.0$, $a_h=0.00$ and pinching hysteresis is considered in Figure 6a, while in Figures 6b-6f one of these parameters is modified each time, in order to illustrate its effect on the system's fragility. A reduction of fundamental period (Figure 6b), an increase of $P-\Delta$ (Figure 6c) or a reduction of ductility (Figure 6d), increase the system's fragility, decreasing its collapse capacity. These effects seem to be larger under short earthquakes, thus trimming the differences between short and long duration. In contrast, an increase of strain hardening ratio leads to an increase in collapse capacity for long and short duration records (Figure 6e). Finally, bilinear systems (Figure 6f) are more fragile under both long and short seismic events and the difference in fragility between these two types of earthquakes is higher than for pinching systems.

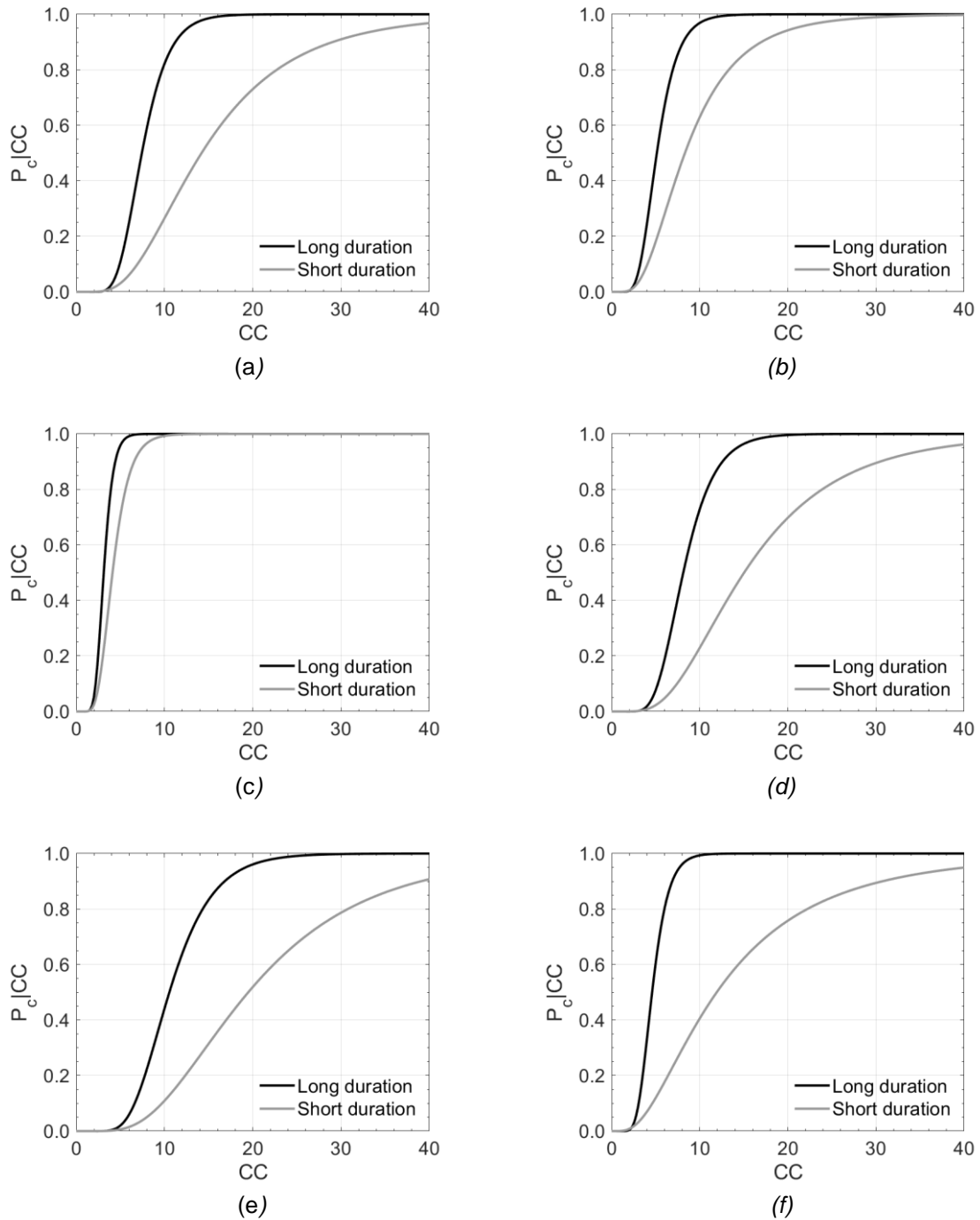


Figure 6. Collapse fragility curves under long and short duration records for a system with (a) $T=3.0s$, $\theta-\alpha=0.02$, $\mu_c=6.0$, $a_h=0.00$ and pinching hysteresis and systems with the following parameters modified: (b) $T=1.0s$, (c) $\theta-\alpha=0.10$, (d) $\mu_c=4.0$, (e) $a_h=0.05$, (f) bilinear hysteresis.

Collapse capacity spectra

In order to provide expressions that allow for the estimation of the collapse capacity of a system with specific characteristics under short and long duration earthquakes, nonlinear regression techniques are applied to the collapse capacities obtained through the numerical analyses. Apart from fitting the numerical data as closely as possible, some additional constraints need to be satisfied.

- The collapse capacities that correspond to non-ductile systems ($\mu_c=1$) need to be accurately recovered by the provided formulae. Therefore, the expressions for collapse capacity spectra of non-ductile systems, which can be found elsewhere (Bravo-Haro *et al.*, 2019), are used as the basis for developing expressions for ductile ones.
- Rigid systems with period almost equal to 0 should yield collapse capacities close to 1 (Tsantaki *et al.*, 2015).

After testing different functional formulae and comparing them by means of statistical indexes, the collapse capacity CC of a system with period T , level of $P-\Delta$ $\theta-\alpha$, ductility μ_c and strain hardening ratio α_h is given as a function of the collapse capacity of the corresponding non-ductile system $CC_{\mu_c=1}$ by the following equation:

$$\frac{CC}{CC_{\mu_c=1}} = 1 + T^{a_1} \cdot (\mu_c^{a_2} - 1) \cdot (\alpha_h^{a_3} + a_4 \cdot \mu_c^{a_5}) \cdot [a_6 \cdot \mu_c \cdot (\theta - \alpha) + a_7 \cdot e^{a_8 \cdot T}] \quad (1)$$

The equations for the collapse capacity of non-ductile systems are provided by Bravo-Haro *et al.* (2019), while the coefficients a_1 - a_8 are given in Table 4, for each hysteresis and duration type.

	Bilinear		Pinching	
	Long	Short	Long	Short
a_1	0.1319	0.2391	0.2734	0.3140
a_2	1.3743	1.3652	0.7271	0.8077
a_3	1.2579	1.0000	1.0000	1.0000
a_4	- 0.0100	- 0.0038	- 0.0029	- 0.0029
a_5	0.0553	1.0000	1.0000	1.0000
a_6	- 1.9407	- 0.3800	- 1.0924	- 0.7138
a_7	3.3050	1.2096	2.9442	2.9085
a_8	- 0.0324	- 0.0531	- 0.1350	- 0.2118

Table 4. Coefficients for collapse capacity spectra.

The resulting collapse capacity spectra for each level of $P-\Delta$ for a bilinear and a pinching system with $\mu_c=4$ and $\alpha_h=0.02$ are plotted in Figures 7 and 8 respectively. As expected, collapse capacities for short duration are plotted above the corresponding ones for long duration earthquakes, while the collapse capacity is scaled down as the level of $P-\Delta$ increases. For low levels of $P-\Delta$, significant differences are observed between short and long duration collapse capacity spectra, indicating the need to include this parameter when estimating seismic collapse.

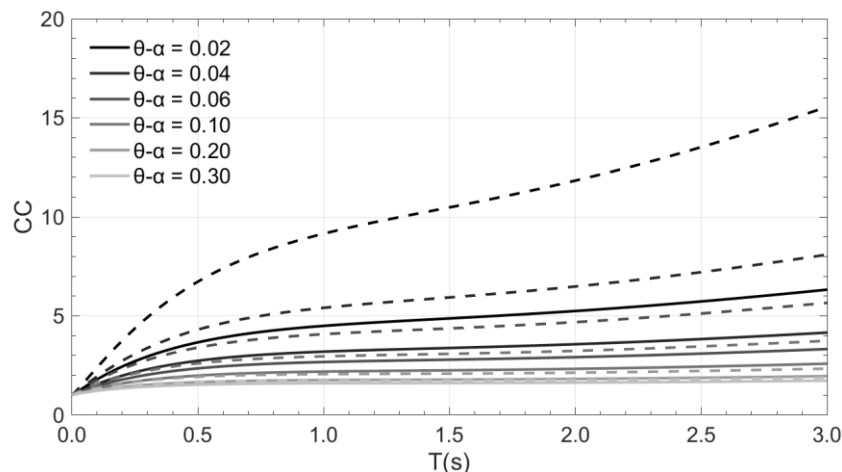


Figure 7. Collapse capacity spectra for long (solid lines) and short (dashed lines) duration for a bilinear system with $\mu_c=4.0$, $\alpha_h=0.02$.

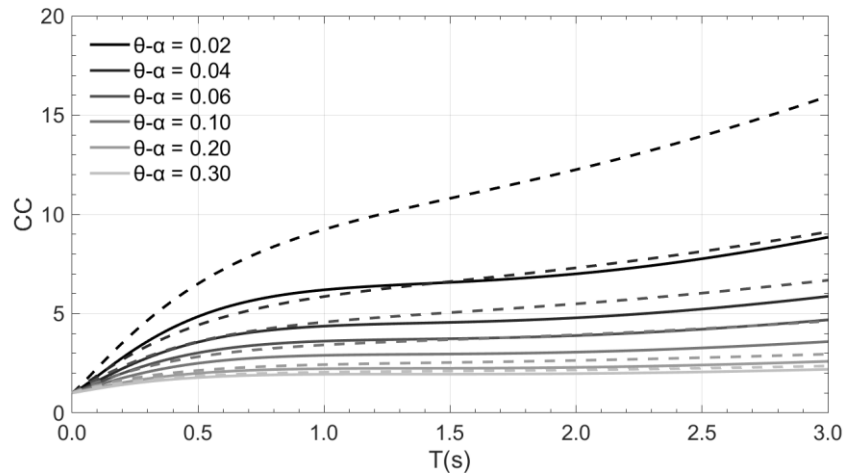


Figure 8. Collapse capacity spectra for long (solid lines) and short (dashed lines) duration for a pinching system with $\mu_c=4.0$, $a_h=0.02$.

Duration effects

In order to quantify the effect of duration, the reduction that it causes in collapse capacity is computed as a percentage of the collapse capacity due to short ground motion records, for each one of the examined SDOF systems. The greatest decrease is discerned for long period systems exposed to small levels of $P-\Delta$, while ductility and strain hardening seem to be less determining factors, as can be confirmed by the following Figures.

Figure 9 shows the reduction in collapse capacity as a function of the last two factors, considering a bilinear and a pinching system with the highest period and lowest level of $P-\Delta$ examined, in which case the greatest amounts of collapse capacity reduction are recorded. It can be noticed that the effect of duration in reducing the collapse capacity is almost unaffected by strain hardening and ductility. Nevertheless, this effect is maximised for a ductility of 4.0 and 6.0 and a strain hardening ratio of 0.00 and 0.10 for bilinear and pinching systems respectively.

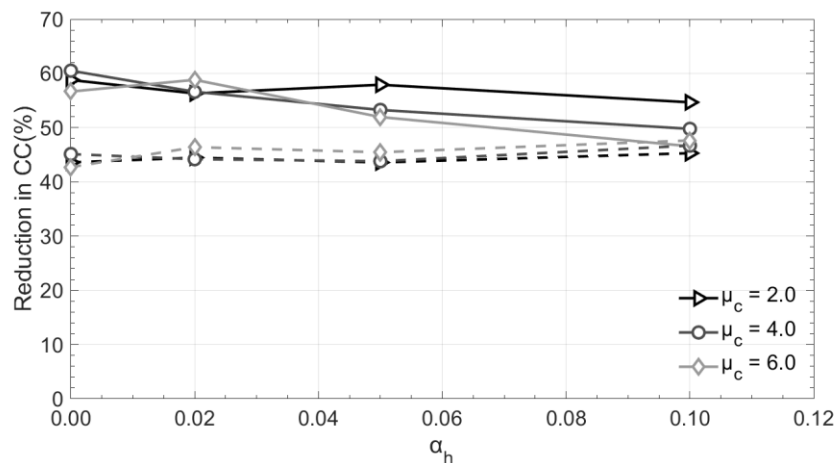


Figure 9. Reduction in collapse capacity due to ground motion duration for a bilinear (solid lines) and a pinching (dashed lines) system with $T=3.0s$ and $\theta-\alpha=0.02$.

These values of ductility and strain hardening are selected to plot the collapse capacity reduction of the two systems as a function of the level of $P-\Delta$ for each examined period in Figures 10 and 11 respectively. In contrast with ductility and strain hardening, the fundamental period and second order effects play an important role in determining whether a system is prone to duration effects, which become increasingly significant as the period increases and the level of $P-\Delta$ reduces. This is because systems with high fundamental period or low levels of negative slope are quite resistant to collapse and can endure many inelastic cycles before their capacity is exhausted. Long duration ground motions tend to impose to them a greater number of cycles compared to

short ones, hence pushing the systems to greater displacements and earlier collapse, which implies lower values of collapse capacity. On the contrary, extremely rigid or vulnerable to second order effects structures tend to collapse almost immediately once they enter the inelastic range without being exposed to many inelastic cycles, and hence, their collapse capacity is close to unity irrespective of ground motion duration.

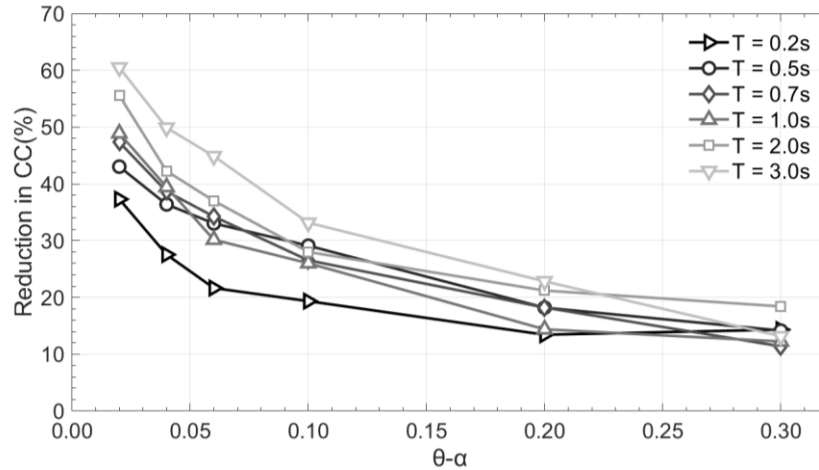


Figure 10. Reduction in collapse capacity due to ground motion duration for a bilinear system with $\mu_c=4.0$ and $\alpha_h=0.00$.

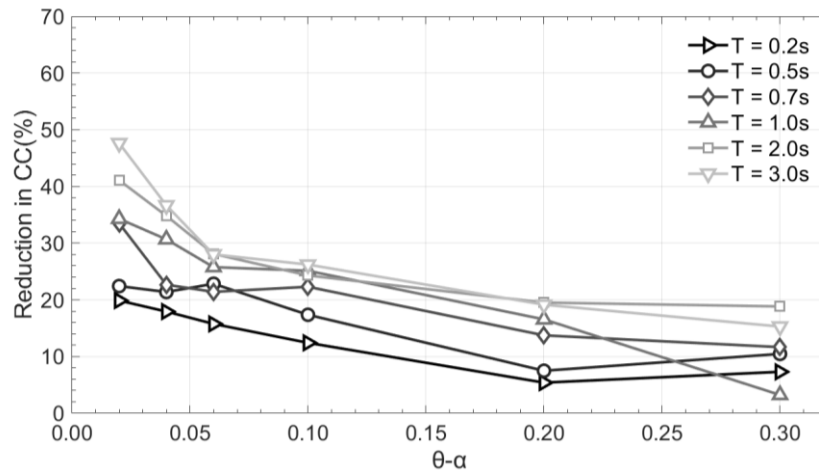


Figure 11. Reduction in collapse capacity due to ground motion duration for a pinching system with $\mu_c=6.0$ and $\alpha_h=0.10$.

It is also worth noticing that duration seems to be more important for bilinear systems, with a reduction up to 61% in collapse capacity, compared to a maximum of 48% in the case of pinching hysteretic behaviour. This is related to the higher collapse capacity of pinching systems due to the part of their inner loops with positive stiffness (Rahnama and Krawinkler, 1993). Longer duration implies that the system is exposed to more inelastic cycles, which are associated with a part of positive stiffness for pinching systems, in contrast to the negative stiffness of the inner loops in the case of bilinear hysteresis. Therefore, the reduction of collapse capacity due to longer duration is lower in the former case.

Conclusions

This study focused on the role of ground motion duration in the collapse assessment of a wide range of ductile, non-deteriorating SDOF systems. The parameters examined included the fundamental period, P- Δ effect, ductility capacity, strain hardening and hysteretic behaviour of the structural systems. As seismic input, long and spectrally equivalent short ground motion records were applied. Strong motion duration was shown to be a major factor influencing the collapse, especially in the case of flexible, bilinear systems subjected to low levels of P- Δ . Such systems

can experience up to more than 60% reduction in their collapse capacity when subjected to longer seismic excitations, indicating the importance of accounting for strong motion duration in collapse estimates. Duration-dependent collapse capacity spectra, such as those provided in this paper, were shown to offer a simple approach for this purpose.

Acknowledgements

The first author would like to acknowledge the support provided by the Skempton Scholarship from Imperial College London and by the scholarship from Onassis Foundation in Greece.

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