

MANAGING UNCERTAINTY IN MODAL PARAMETER ESTIMATION FROM AMBIENT VIBRATION MEASUREMENTS

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Abstract: For massive, long or tall structures where forced vibration testing is unfeasible, operational modal analysis of ambient response data is used to estimate modal parameters (mode frequency, damping and shape). Estimates have usually been reported without tolerance intervals so reliability is questionable, and the best way to obtain the most reliable estimates is only understood empirically. A recent EPSRC project has developed the technology of Bayesian operational modal analysis (BAYOMA) to provide qualitative guidance for modal tests in terms of choice of sensor (what noise floor) number of measurement points (complexity of the test setup) and duration (for a single estimate, trading off the effects of non-stationarity). The uncertainty law for damping ratio is explained and examples given of application, for two structures. The results are particularly important for the case of supertall buildings where vibration serviceability governs design and where design thus depends critically on reliability of available information, particularly of damping ratio.

Introduction

The paper summarises research on Bayesian operational modal analysis (BAYOMA), a methodology for identification of modal parameters (MPs) from ambient vibration data that encapsulates the uncertainty in such a way that it is possible to design field experiments to achieve a desired level of uncertainty. The most obvious direct application is to extreme long span and tall structures (LSTS) whose performance depends strongly on modal parameters (MPs); the ability to manage uncertainty in MP estimates that feed into design has implications for risk management.

Increasingly MP estimation is a requirement for structural commissioning tests to corroborate design, and vibration tests on LSTS have traditionally had to rely on MP estimation from ambient vibration measurements. Operational modal analysis (OMA) techniques such as stochastic subspace identification (SSI) and frequency domain decomposition (FDD) (Peeters & De Roeck, 2001; Brincker, Zhang, & Andersen, 2001) are increasingly popular for such purposes but there is a long association of OMA with unreliable estimation (Kareem & Gurley, 1996; Pagnini, 1998) compared to estimates obtained from logistically complex and often unfeasible forced vibration tests. Methods for calculating the potential variability of MP estimates due to variability of data have been developed, e.g., for SSI (Mellinger, Döhler, & Mevel, 2016; Pintelon, Guillaume, & Schoukens, 2007; Reynders, Pintelon, & De Roeck, 2008; Döhler, Lam, & Mevel, 2013) but these methods offer no insight.

Bayesian OMA methods (Au, Zhang & Ni, 2013) adopt a radically different philosophy to previous OMA procedures such as half power bandwidth and SSI respectively by posing the question 'what is known about MP based on the given data?'. Answers are expressed via the *posterior* probability density function (PDF) of MP implied fundamentally by Bayes' theorem (Beck, 2010). The covariance matrix associated with the posterior PDF then naturally quantifies the identification uncertainty of MP for the given data set and modelling assumptions. This is in contrast to non-Bayesian methods where the uncertainty refers to the estimates of the MP rather than the MP itself, and is 'inherent' in nature, i.e., does not depend on data.

The BAYOMA approach is described in (Au, 2017). The PDF for the value of a spectral estimate at a (FFT) frequency line can be constructed for given MPs in a system and a contribution of noise. The likelihood of obtaining a specific FFT (signature) for given MPs and noise is a product

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of all these probabilities. BAYOMA turns the problem around to identify the most probable value (MPV) of the MPs *given the measured FFT*, along with the uncertainty.

BAYOMA does so by minimising a likelihood function (of MPs for given data). The minimum finds their MPV while the curvature at the minimum, expressed as a Hessian matrix, provides the uncertainty (as a coefficient of variation or COV) via its inverse.

Algorithms for calculating MP uncertainty, Bayesian or non-Bayesian, only allow COVs to be calculated (in a ‘point-wise’ manner) when the data are given but they don’t say anything about why the uncertainty takes such a value or how it can be controlled other than empirically, by observation. For example, if ambient response data are obtained over an infinitely long period during which time the system properties and spectrum of the input remain the same (stationary), then it can be observed that standard deviation of values collected from different data segments reduces with the square root of the duration of those segments. Other conditions may have an impact, for example the quality of the sensors used, the number and locations of the sensors and the character of the loading. A particular challenge authors have met is where structures exhibit ‘close modes’ i.e. very similar frequencies and very similar mode shapes, for example in an office tower with polygonal section such as an octagon or a square. The question is, how to make the most of a field measurement opportunity with restricted access to a site over a short time window, a limited number of sensors (not always the best quality) and various noise sources, in order to achieve the most reliable MP estimates?

The EPSRC BAYOMALAW research project (summarised in Figure 1) aimed to address this question, delivering ‘uncertainty laws’ using Bayesian inference to express MP uncertainties explicitly as functions of test configurations such as measurement noise, environmental load intensity and sensor configuration. The main objectives were to capture the effect of signal/noise ratio (SNR), close modes and multiple setups having more degrees of freedom (DOFs) than sensors i.e. to relate MP uncertainty directly to test configuration so that it could be *prescribed and managed*.

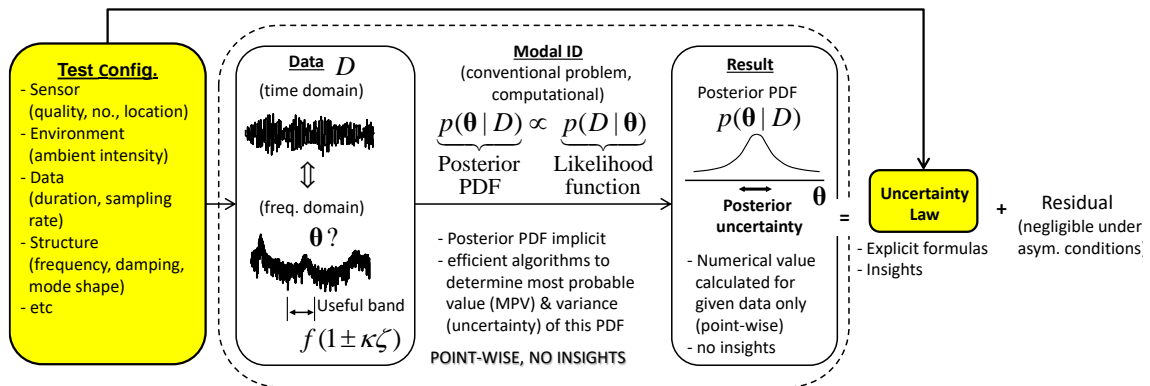


Figure 1: Objective of BAYOMALAW project

This paper shows how knowledge of experimental conditions can help plan an ambient vibration tests on a type of structure where damping ratio is notoriously difficult to estimate (a long span bridge) and then shows how BAYOMA process is applied to identify a set of modes. Subsequent to the bridge test, a different challenge for BAYOMA was identified, which is identification of mode shapes in an axisymmetric structure (a lighthouse) having very close natural frequencies.

BAYOMALAW and damping

Damping ratio is the MP that has the highest uncertainty. For well-separated modes, small damping and long data in a single setup, the posterior COV d (coefficient of variation = standard deviation/mean) is asymptotically given by

$$\delta = \delta_0 \sqrt{1 + a(\kappa)/\gamma} \quad (1)$$

where ‘ \sim ’ (read as ‘asymptotic to’) denotes that the ratio of the right-hand side to left hand side tends to 1 under the stated asymptotic condition ($\zeta \rightarrow 0$, data length $\rightarrow \infty$).

$$\gamma = S/4S_e\zeta^2 \quad (2)$$

is the ‘modal SNR’, the ratio of acceleration data modal PSD ($S/4\zeta^2$) to the signal noise PSD (S_e) at the natural frequency and

$$\delta_0 = 1/\sqrt{2\pi\zeta N_c B(\kappa)} \quad (3)$$

is the ‘zeroth order’ law that gives the COV when the modal SNR is infinite. This captures the empirically observed relationship of COV with data duration since $N_c=Tf$ is the data duration (T) expressed as a multiple of the natural period ($1/f$) of the identified mode. The two parameters $a(\kappa), B(\kappa)$ are functions of a dimensionless ‘bandwidth factor’ that reflects usable bandwidth $f(1\pm\kappa\zeta)$ around the natural frequency f ($\kappa=1$ gives the ‘half-power bandwidth’) without incurring significant modelling error. $B(\kappa)$ is exponentially asymptotic to unity for high κ and $a(\kappa)$ increases linearly with κ . Data duration is considered long when the number of FFT lines N_f in the usable band is large compared to 1. This number can be reasoned to be equal to $N_f = 2\kappa\zeta N_c$.

The effects of the test configuration (first order correction) are fundamentally quantified in terms of γ (modal SNR) since sensor and data acquisition hardware affect the noise PSD S_e near the natural frequency whereas the modal force PSD S is affected by the number and location of sensors, leading to the principle (for a single mode) of maximising the sum of squares of mode shape values when taking a measurement.

During the BAYOMALAW project, several structures were tested to validate the uncertainty laws as well as to evaluate and enhance the performance of the BAYOMA procedure in demanding conditions. Structures included Jianguyin Bridge (China), Rainbow Bridge (Tokyo), a tall building in Canary Wharf (London) and a number of offshore lighthouses. The Jianguyin Bridge study is described in detail since it included a planning stage applying the uncertainty law given above.

BAYOMA and damping: Ambient vibration test of Jianguyin Bridge

Jianguyin Bridge (Figure 2) has a single 1385m suspended span and was completed in 1999. The 0.876m diameter main cables each support the deck via 85 pairs of vertical hangers, and back stays carry which carry no deck load have a diameter of 0.897m. The main girder is a streamlined box section 36.9m and 3m high carrying a 29.5m wide deck with three lanes of cars, buses and trucks in each direction. There is a 1.8m walkway either side of the deck with limited access, for maintenance purposes, and towers comprise two 190m hollow tower reinforced concrete columns and three hollow transverse beams are accessible by lifts from roadway level.



Figure 2: Jianguyin Bridge

Planning with preliminary data

Planning an ambient vibration test, particularly overseas, requires extensive planning to achieve clear objectives. In this case the aim was to identify natural frequencies, damping ratios and mode shapes of the first few lateral, vertical and torsional vibration modes, up to approximately 1Hz. Full identification of all modes, in an exercise as extensive as the 2008 Humber Bridge test (Brownjohn, Magalhães, Caetano, & Cunha, 2010) would require access to both sides of the deck and to all four tower columns. Such access would be more challenging for Jianguyin and since the focus was on the deck modes, with some means of identifying which modes were torsional, limited measurements in the tower and partial measurements on a second side of the deck would suffice. Expecting to have only three days available required a plan on sensor locations and measurement durations informed via equations 1-3. Hence approximate values of lowest natural frequency and damping ratio as well as sensor signal to noise ratio would be needed. A sample of data from the partially operational Jianguyin Bridge SHM system, was available indicating a vertical mode at 0.106Hz with a damping ratio of 4% and a usable identification band of [0.07 0.11]Hz with a bandwidth factor $\kappa=4.78$. Figure 3a shows that using equation (3) for 15% CoV, $N_c=360$ natural periods are required, corresponding to one hour for the given mode.

From the preliminary data, $S \sim 9 \times 10^{-8} \text{g}^2/\text{Hz}$ and $S_e \sim 5.3 \times 10^{-7} \text{g}^2/\text{Hz}$, which for the first vertical mode provides an estimated $\text{SNR}=27$ using equation (2) and 4% damping. Compared to the noiseless situation (0^{th} order estimate of δ_0) this SNR leads to a 20% increase as indicated in Figure 3b. By optimising the sensor locations and using sensors with a lower noise level in the main test, it would be expected that a larger SNR would be achievable. Data for lateral response were not available; typically the first lateral mode frequency could be half the first vertical mode frequency, and more challenging to identify.

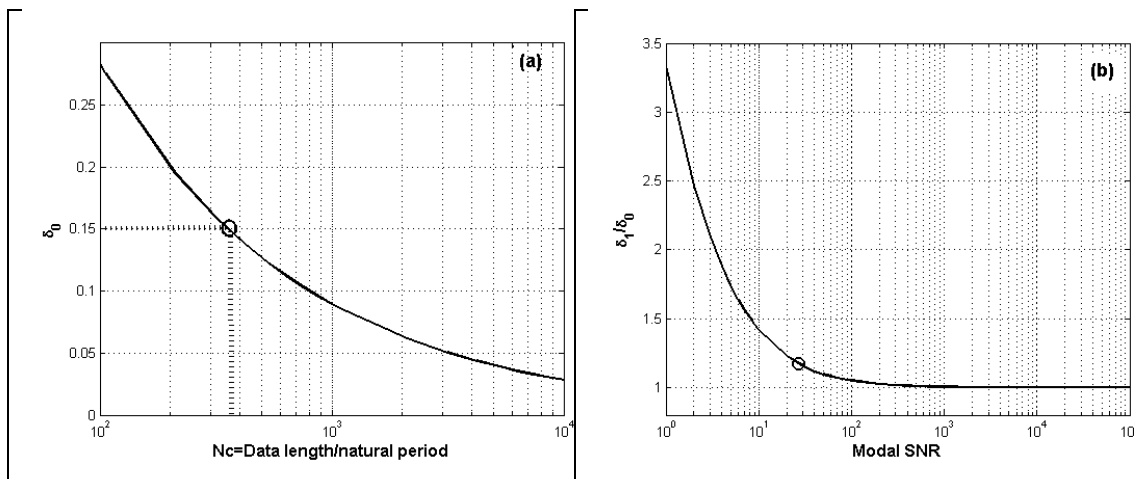


Figure 3: 0^{th} and 1^{st} order COV for damping ratio given parameters for Jianguyin Bridge first vertical mode. (a) 0^{th} order without effect of SNR, (b) 1^{st} order correction depending on SNR

Planning without preliminary data

If no prior information were available then MPs for Jianguyin Bridge might be expected to be similar to those obtained for a similar bridge. A close analog is Fatih Mehmet Sultan Bridge, the second bridge over the Bosphorus or ‘B2’ (Brownjohn, Dumanoglu, & Severn, 1992), for which MPs were estimated as $f=0.125$ Hz, $\zeta=1.33\%$ for first vertical and, $f=0.077$ Hz, $\zeta<14.4\%$ for first lateral.

Without sample data the bandwidth factor κ is not known but since a noisier signal buries the spectral peak, reducing the useful frequency range of information, a reasonable choice consistent with common practice is $\kappa=2\sqrt{\gamma}$.

Although a value of $\text{SNR}=27$ is available from preliminary data, a typical value could also be obtained using the noise floor of the measurement system to be used in the test (checked in the lab to be $\sqrt{S_e}=0.03 \mu\text{ms}^{-2}/\sqrt{\text{Hz}}$) and modal acceleration from B2 first lateral mode ($\sqrt{S}/2\zeta=5 \text{ mms}^{-2}/\sqrt{\text{Hz}}$), giving a worst case $\text{SNR}=3 \times 10^4$, actually much more reassuring than the preliminary data, so that the measurement duration, assuming 4% damping could be less than $N_c=360$, i.e. one hour for 0.1 Hz mode for 15% COV -which is actually quite low considering historical unreliability of damping estimates. Considering the need to measure the lateral mode, 3600s recordings were planned.

Ambient vibration test: Jiangyin

The AVT took three days, from 25th to 27th April 2017. A set of four bespoke wire-free loggers using oven-controlled crystal oscillators (OCXOs) and National Instruments CompactRIO hardware, 12 QA-750 accelerometers, 240 m of 10 m length colour-coded signal cable were flown from the UK to Shanghai, with high-capacity batteries provided locally. Weather was fine and most measurements were made on the east walkway which was accessed by driving from the south and parking briefly in a traffic lane at the south tower. Hanger attachment points were used as potential ‘test points’ or TPs, measuring at every fourth hanger. A master OCXO box was used as a reference with lateral (transverse direction) and vertical accelerometers at TP67 and TP71, aiming to identify mode shape pieces using three roving OCXO boxes, each at a single TP, except for the limited measurements in the south tower and a single measurement on the west walkway. Figure 4 shows a typical measurement setup.

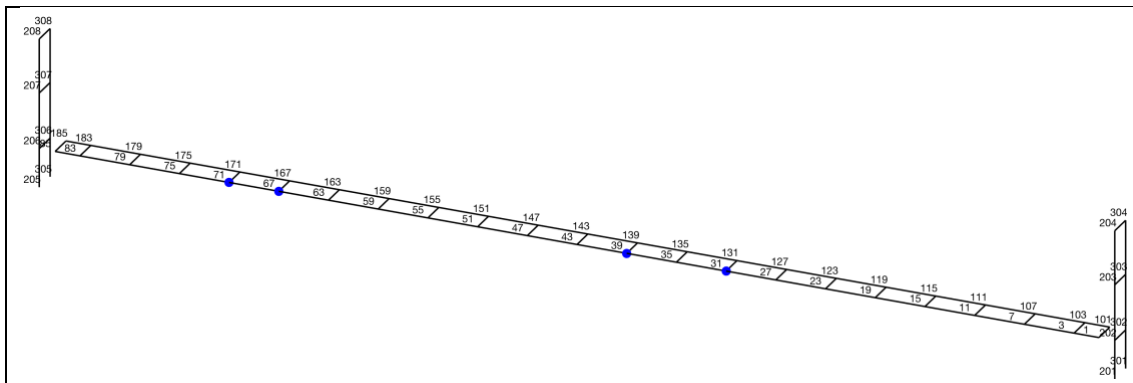


Figure 4: Jiangyin Bridge test points and sensor locations for a measurement setup

A detailed method statement and risk assessment was prepared for a sequence of setups with one-hour recordings (based on the 360×Nc requirement). Figure 5 shows the loggers being synchronised at the start of the sequence and the pair of accelerometers at TP71.

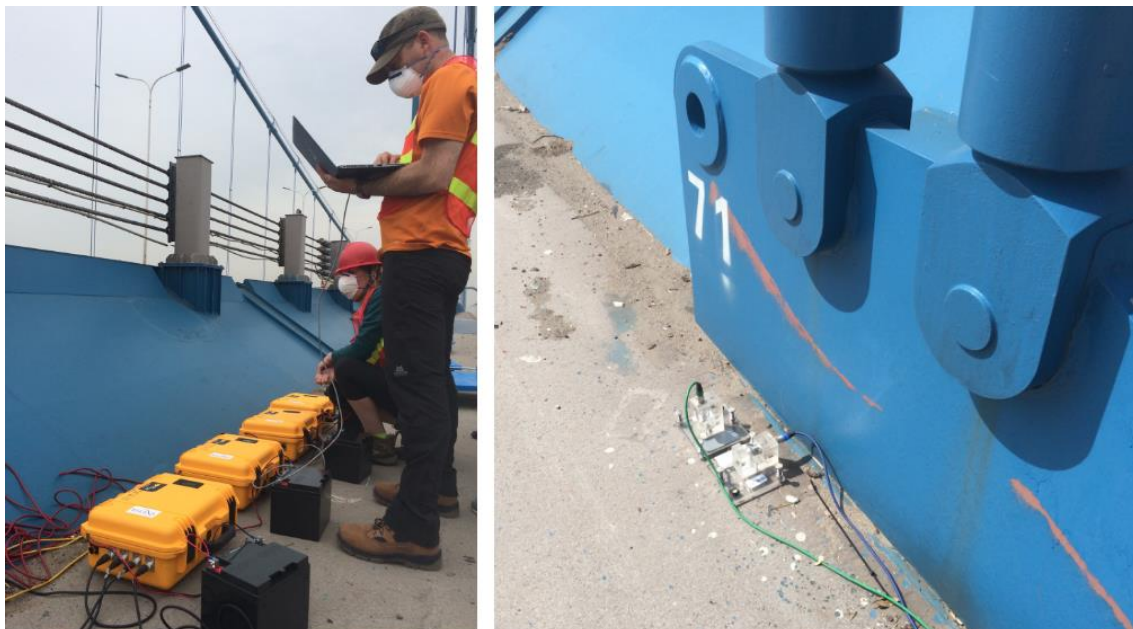


Figure 5: Setting up acquisition for initial huddle test and reference accelerometers at H71

The reference accelerometer signals were recorded by the master logger continuously during the day of measurement while the three slave loggers were roved to remaining TPs. The moves were coordinated by mobile phone text messages and timed so as to maintain a minimum one hour recording for all loggers. The final setup was designed to characterise the tower component of deck girder modes as well as modes only involving the tower. The tower was accessed at roadway level and the middle portal was accessed by a lift. Access to the upper portal was via transfer to

a second lift at a mezzanine level and to the top of the portal by staircase. For the internal measurements the value precision synchronisation without GPS was proven.

Modal parameter estimates using Bayesian operational modal analysis

The square root power spectral density (PSD) of the first measurement setup recording response at H67,71,83 and 75 in the lateral and vertical directions is shown in Figure 6. The four lower curves represent the much weaker lateral response, a situation typical for long-span bridges. Usually (as in the lighthouse example described later) the singular value spectra provide clear indications of modes to identify, but in this case the PSD spectrum is clearer. The initial guesses of natural frequency and the selected frequency bands for modes of interest are indicated in the figure with symbols ‘o’ and ‘-], respectively with ‘*’ denoting that a mode identified using data in the lateral direction only. Most lateral modes are identified this way.

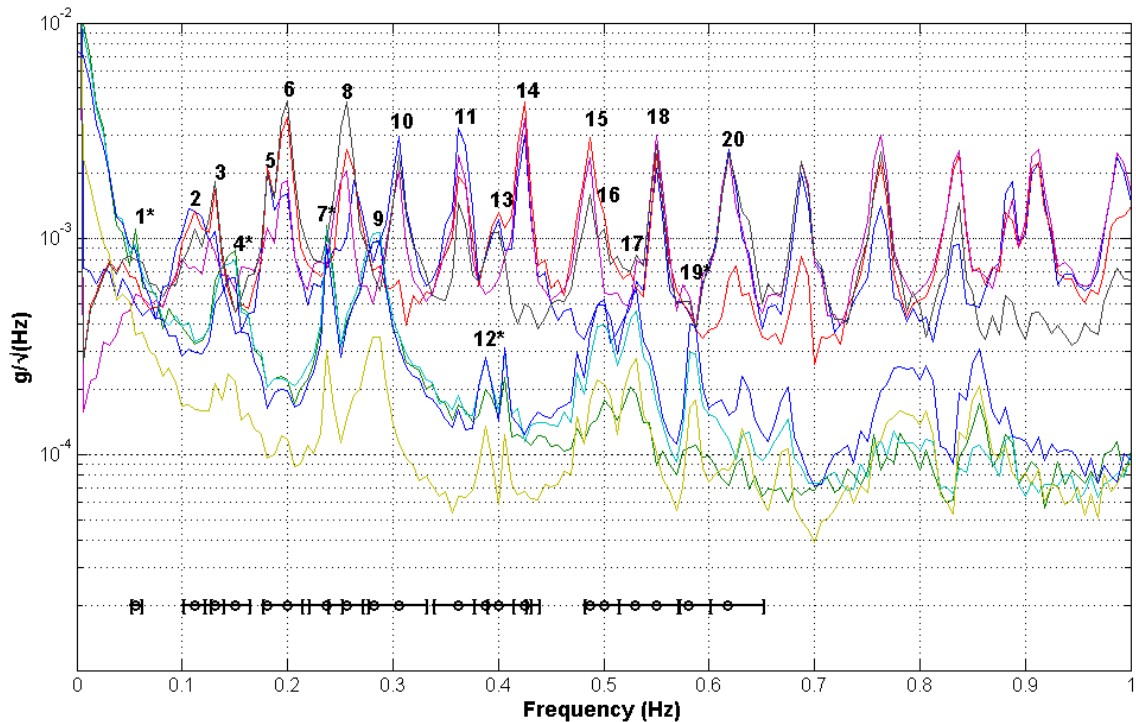
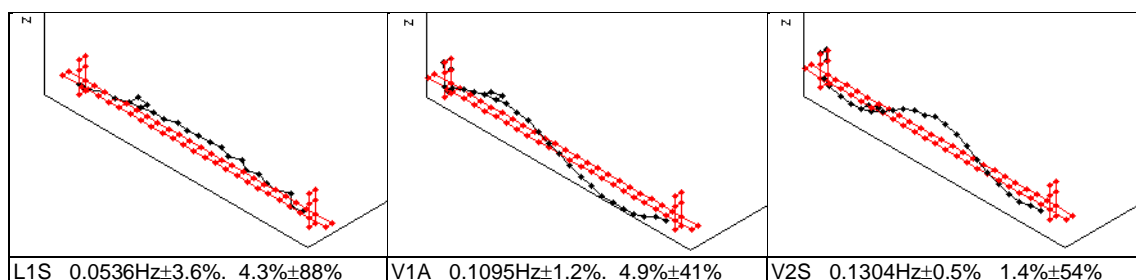


Figure 6: Square root PSD spectrum of setup 1

Modal parameters were identified in individual setups using the FFT-based BAYOMA procedure (Au, 2012a, 2012b), and the overall mode shapes assembled using the global least square method (Au, 2011). In this procedure only the real parts (or projections on to real axes) of the mode shape ordinates are used.

Figure 7 shows the assembled global mode shapes for the first two lateral, vertical and torsional modes. The first mode (L1S) is a symmetric lateral mode with a natural frequency of 0.0536Hz although there is an increase of the PSD towards 0Hz which derives from a combination of measurement noise and quasi-static rotation which together almost bury the modal contribution. Absence of lateral mode peaks in the vertical response caused convergence problems in the BAYOMA procedure (which is looking for the maximum of a likelihood function) when all the data channels are involved for inference. Hence the overall mode shape is assembled assuming zero mode shape values in the vertical direction.



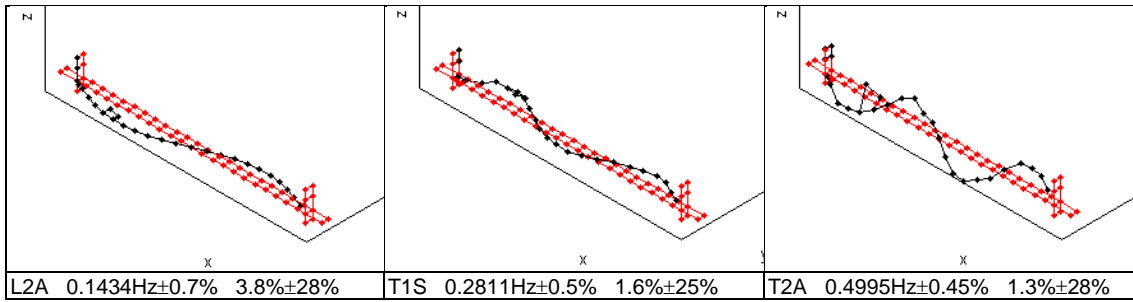


Figure 7: Sample of identified global mode shapes

Figure 8 shows the ratio δ_1/δ_0 (the first order effect of equation 1) for damping estimation of L1S and V1A for six of the measurement setups. The curves show the effect of SNR on COV, the circles denote the value for each setup as predicted by uncertainty laws and the %ages are the COV values. The figure shows that even if SNR is hugely increased, no better than 30% improvement in COV could be obtained from response data.

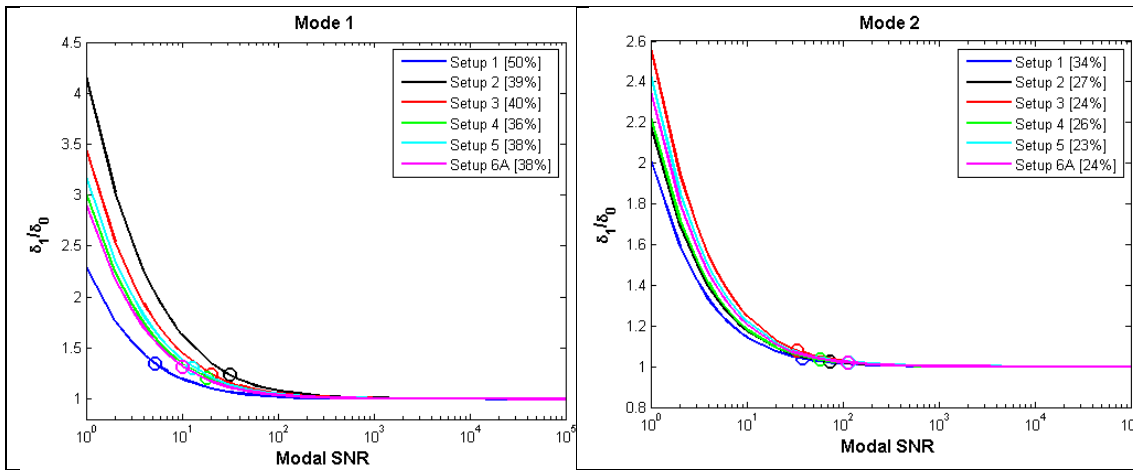


Figure 8: SNR effect with damping COV for mode L1S (left) and V1A (right)

Vibration monitoring in lighthouses: Wolf Rock and Eddystone

A different challenge in OMA was observed in a structure having very closely spaced modes. The particular example studied here is Eddystone (Figure 9), in southwest UK.

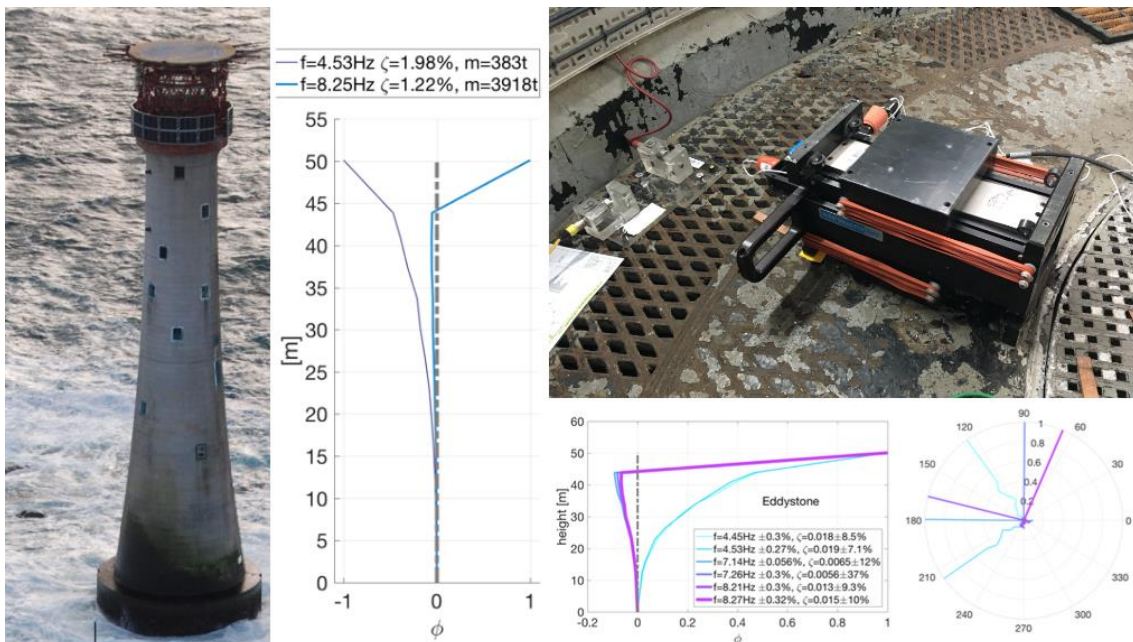


Figure 9: Eddystone Lighthouse: mode shapes from forced and ambient testing.

Eddystone comprises elliptically a tapered masonry tower formed from pre-shaped granite blocks keyed, grouted and bolted to each other and the foundation and topped with a helideck assembled on site from pre-formed steel frames and a horizontal steel grillage landing pad. A forced vibration test was carried out in order to estimate mode frequencies, damping ratios, mode shapes and modal masses. Figure 9 shows two mode shapes, having the same shape in the masonry tower and opposite phase in the helideck, obtained using shaker testing. The shaker was set alternately in two orthogonal directions chosen for logistical convenience, and detailed examination of the frequency response functions indicated presence of two very close modes, although no information was available on mode shape azimuth (compass bearing).

To address this, ambient vibration measurements were used, with modal identification using BAYOMA which revealed the mode orientation, as shown in Figure 9 (bottom right). While the uncertainties in the frequency and damping estimates for short measurements (approx. 30 minutes) are indicated, uncertainty in the mode shape is not, and it is strange that while the shapes are approximately at right angles in the pairs (three pairs, including the two obtained from the shaker test) do not all align. The six mode (two pairs) are shown clearly in the SVD plot (Figure 10) showing that the frequencies in the highest pair are barely distinguishable.

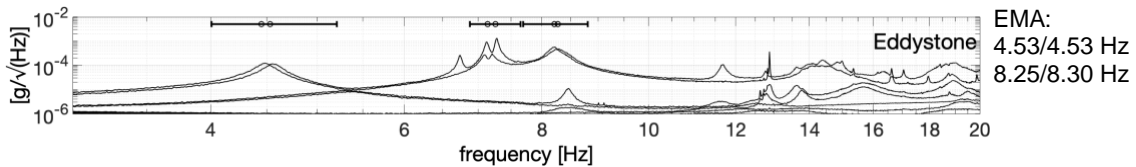


Figure 10: singular value decomposition of Eddystone Lighthouse PSD spectra

From an extended overnight measurement, with an array of biaxial accelerometer pairs throughout the lighthouse, including the helideck, the variation and uncertainty of mode shapes was tracked by identifying the the three mode pairs in successive 5-minute frames. Damping values are not important for this study; the (academic) interest is in the mode shapes and how their variation and identification uncertainty relates to closeness of the modes. The first column in Figure 11 shows frequency estimates and $\pm 1\sigma$ bounds as error bars, the second column shows the mode shape angles as circles with radii proportional to σ . The right-hand column shows the most probably mode shape values in plan to indicate the variability among estimates. Contrast the mode shape variance for modes 5 and 6 and frequency closeness with that for modes 3 and 4. Apparently closer frequencies generate more uncertainty in mode shape angles, which seems intuitively correct.

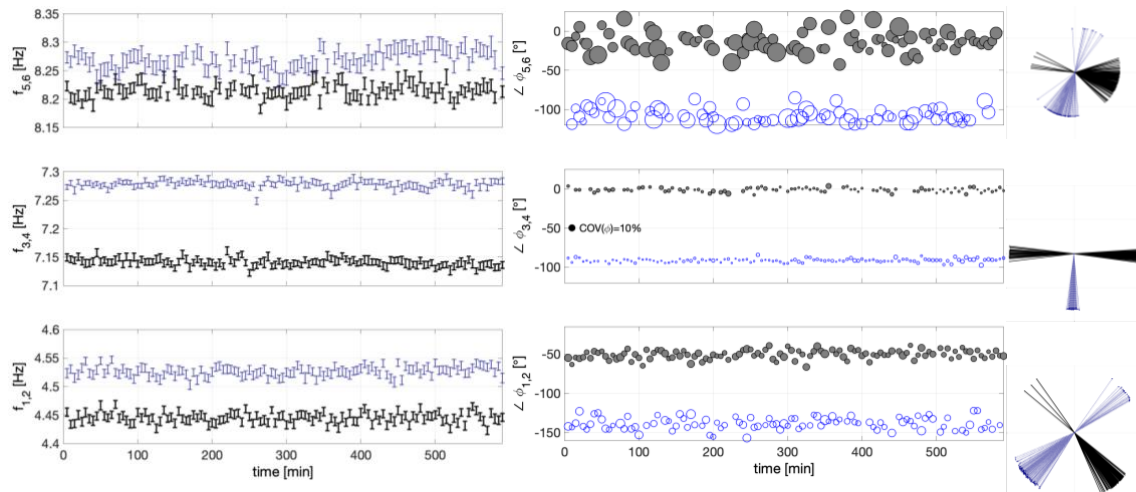


Figure 11: BAYOMA identification of Eddystone first three mode pairs for 5-minute frames

Next consider identification using 20-minute frames. Variance errors approximately halve, consistent with having four times as many time series data points.

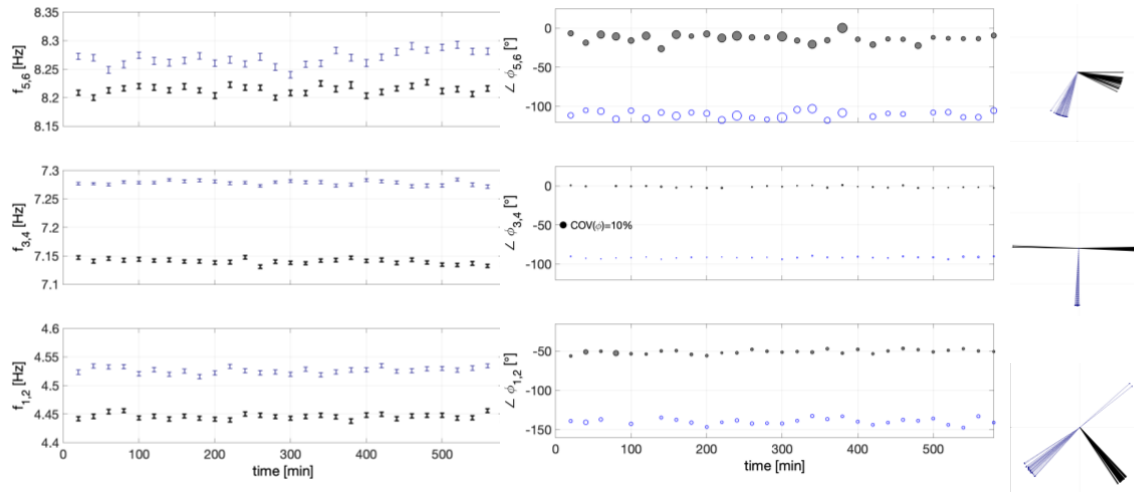


Figure 12: BAYOMA identification of Eddystone first three mode pairs for 20-minute frames

Uncertainty laws for mode frequency, damping and shape estimates for close modes are being developed in the final phase of the research.

Summary

Bayesian operational modal analysis remains an unusual approach for identifying modal parameters obtained from ambient vibration measurements yet it provides the capability to offer insights into the origin of uncertainty depending on the experimental conditions and the mature of the modal properties. As such it is less of a ‘black box’ than traditional (currently popular) methods such as stochastic subspace identification. BAYOMA enables control of identification uncertainty which has implications for application of ambient vibration methods in design, model validation and structural health monitoring, among other applications.

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