THE IMPORTANCE OF CONSIDERING BEYOND DESIGN BASIS RESPONSE OF NUCLEAR PLANTS TO EARTHQUAKES

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Abstract: This paper provides a very simplified analysis of the risk arising from earthquake hazard on nuclear plant. The analysis captures the effect of the hazard being defined as an exceedance probability, rather than a point probability, per unit time. It also investigates the common cause nature of the hazard to multiple lines of protection and illustrates the dramatic effect this feature can exert on plant risk. One of the principal findings from the analysis is that the potential for plant failure due to earthquake hazard is most likely between the design basis value and twice this value, if the design basis represents a 1% conditional probability of plant failure. This highlights the importance of analysing the beyond design basis response of nuclear plant.

This analysis is presented as part of a discussion of the changes made recently to the Office for Nuclear Regulation's (ONR's) Safety Assessment Principles (SAPs). These changes were prompted by lessons arising from the Fukushima event in March 2011 and include an extended discussion of beyond design basis response.

INTRODUCTION

Following the Great Japan Earthquake of March 2011 and especially the effect of this event on the Fukushima Dai-ichi nuclear reactors, operators and regulators around the world embarked on a substantial re-evaluation of the basis for safety against natural hazards at nuclear sites. The UK nuclear industry has invested heavily in this post-Fukushima work, especially in improving the resilience of such sites to extremes of these hazards.

The Office for Nuclear Regulation (ONR) is the principal nuclear regulator in the UK and has been fully engaged in overseeing this work (ONR 2011). One of the most important regulatory tasks has been to update the ONR's Safety Assessment Principles (SAPs) that ONR staff use to judge the adequacy of nuclear safety at nuclear licensed sites in the UK (ONR 2014). This paper provides a summary of these changes as they affect consideration of external hazards generally, but concentrates on earthquake.

Nuclear plants are designed to withstand (or be assessed against) hazard levels known as design bases. In the UK, for natural hazards such as earthquake, external flooding and extreme weather, these are generally a conservative estimate of the event that could be exceeded once in 10,000 years (see ONR (2014) para. 239). The ability of a plant to withstand hazard levels beyond the design basis is often expressed as a beyond design basis margin. The revised SAPs provide greater detail on what the regulator expects such an analysis to deliver, which in summary are:

a) Confirm the absence of "cliff edge" effects just beyond the design basis.

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b) Identify the hazard level at which safety functions could be lost (i.e. determine the beyond design basis margin).
c) Provide an input to probabilistic safety analysis to demonstrate that risk targets are met.
d) Ensure that safety is balanced so that no single type of hazard makes a disproportionate contribution to overall risk.
e) Provide an input to severe accident analysis.

The paper will use a very simplified model of a nuclear plant to explore how beyond design basis margins affects the overall seismic risk. The calculated results will illustrate why ONR considers the analysis of beyond design basis capability to be an important part of demonstrating that nuclear plant meet the intent of the SAPs.

The plant model can accommodate both one and two lines of protection against seismic faults. A number of worked examples are provided to illustrate:

- How such analysis can be used to examine compliance with risk criteria.
- How design basis selection and beyond design basis margins can influence seismic risk.
- The influence in risk terms of moving from one to two lines of protection.
- The implications of characterising seismic hazard by a full exceedance frequency hazard curve, rather than just considering the design basis value and beyond design basis margin in isolation.

Conclusions are drawn from the worked examples to draw out some general conclusions about how the traditional design basis approach to demonstrating nuclear safety to seismic hazard influences the plant seismic risk.

BEYOND DESIGN BASIS ANALYSIS AND DESIGN BASIS MARGINS

In the UK, the safety of nuclear plant subject to external hazards is typically analysed by three complementary approaches, although the extent to which each is pursued in any particular case depends on the hazard, the plant complexity and the potential consequences of failure and its history: Design Basis Analysis (DBA), Beyond Design Basis Analysis (BDBA) and Probabilistic Safety Analysis (PSA). This paper concentrates on the role of BDBA.

Beyond Design Basis Analysis is often a natural extension of DBA, where obvious conservatisms are removed from the design basis plant response model to give an indication of the plant's "best estimate" response to the design basis hazard. BDBA therefore predicts that the plant response will be stronger and more resistant to hazard induced design loads than the DBA and implies that it will fail at higher hazard levels, but with less confidence that required safety functions will be delivered at these levels.

The objective of a BDBA is to demonstrate that:

- There are no cliff edges in plant response such that there is a disproportionate increase in accidental consequences just beyond the design basis.
- No additional significant faults are initiated just beyond the design basis.
- If successful, the BDBA demonstrates that the plant is very robust to the Design Basis hazard challenge, and accounts for analysis and data uncertainties in both the hazard definition and the plant response to the hazard.

Noting the difference between plant response to a seismic event computed from a DBA and BDBA, a seismic margin can be defined as follows.

Let *PM* represent the plant model and x be the measure of hazard severity, generally taken as the peak ground acceleration (PGA) in the free field local to the site, $H(x_{DB})$ is the probabilistic hazard function at the design basis(DB) level, x_{DB}. If the plant response to this hazard is *R*[4], then symbolically we can write:

DBA	$R(x_{DB}) = [PM_{DB}] \times H(x_{DB})$
BDBA	$R'(x_{DB}) = [PM_{BDB}] \times H(x_{DB})$

where $R' < R$. This allows a margin to be defined either qualitatively in terms of the difference between the plant models[5], $[PM_{DB}] - [PM_{BDB}]$, or quantitatively in terms of the difference between the responses, $R(x_{DB}) - R'(x_{DB})$. These are entirely reasonable ways of defining a margin, but they only measure the difference in response in terms of the plant models used in both analyses; the hazard itself makes no contribution to the margin definition except as a constant scale factor.

An alternative approach, adopted here, is to use a compound definition of margin that accommodates both the difference between the plant models and the effect of the hazard function on plant response as the plant model changes and is able to resist an increased hazard level, x, for the same response. Assume a beyond design basis (BDB) hazard level, x_{BDB}, such that $R'(x_{BDB}) = R(x_{DB})$, then we can construct a quantitative margin as follows:

$$\Delta = x_{BDB}/x_{DB}, \tag{1}$$

Where a quotient definition, rather than a difference by subtraction, is preferred since this automatically normalises the margin and enables comparison of margins at different design basis hazard levels.

SEISMIC HAZARD, FRAGILITY AND RISK

Seismic risk: The consequential risk from plant faults initiated by seismicity can be expressed as

$$Risk = P(\infty) \times C \tag{2}$$

where both terms on the RHS are probabilities, the first covering plant failure for a given fault and the second covering the consequential effects of such failure in terms of radiation dose or number of fatalities exceeded etc. In common with many similar analyses, we assume here that once failure has occurred the ability to deliver a given consequential effect (in the UK this might be a Target 8 dose to the public) is "guaranteed", so that we can set the consequence term to unity. The seismic risk from a given fault is therefore just the probability of plant failure under this fault condition, $P(\infty)$. The total seismic risk is therefore the sum of these values over all faults initiated by seismic hazard.

We expand $P(\infty)$ in terms of a hazard function and a plant fragility function according to the standard equation (see e.g. Kennedy (2000)):

$$P(\infty) = \int_0^\infty H(x) \cdot f(x) \cdot dx \tag{3}$$

$H(x)$ is the seismic hazard function with peak ground acceleration (PGA) parameter x, and f is the probabilistic fragility density function defining the plant response to the fault, with cumulative function F. Eqn. (3) is appropriate when applied to a single Line of Protection (LoP) against the fault. We will be interested here in the effects of hazard initiated faults on

[4] *R* is assumed to be a plant response that is closely aligned to a safety function.
[5] This might typically include removing one or more conservative assumptions from the DBA plant model to form the BDBA model.

plants with two independent lines of protection since this is a more realistic scenario. For maximum safety benefit, these should be organised as independent, uncorrelated lines of protection that both have to fail to generate loss of safety function leading to a consequential effect. The two lines of protection are then said to form an ANDed combination, and we can develop an appropriate risk equation from eqn. (3) as

$$P_{A\cap B}(\infty) = \int_0^\infty H(x)\cdot f_{A\cap B}(x)\cdot dx = \int_0^\infty H(x)\cdot [f_A(x)F_B(x) + F_A(x)f_B(x)]\cdot dx \quad (4)$$

where A and B refer to independent, uncorrelated lines of protection.

Seismic Hazard Model: In this paper we make use of an extremely simple power law model developed for seismic hazard by Kennedy (Kennedy 2000)

$$H(x) = h_0 \cdot x^{-n} \quad (5)$$

where h_0 and n are parameters defining a particular hazard and x is peak ground acceleration (PGA) as before. This hazard model is expressed in frequency rather than probability terms for simplicity, but at the low hazard frequencies relevant here (< 10^{-2}/yr), the numerical difference between probability and frequency descriptions is negligible and does not warrant the increased mathematical complexity needed to handle a probabilistic approach. Figure 1 shows the model with parameters to give $H(0.25\text{g}) = 10^{-4}$/yr, a typical value for a "high" seismicity UK site

$$H(x) = 6.113\cdot 10^{-7}\cdot x^{-3.677}$$

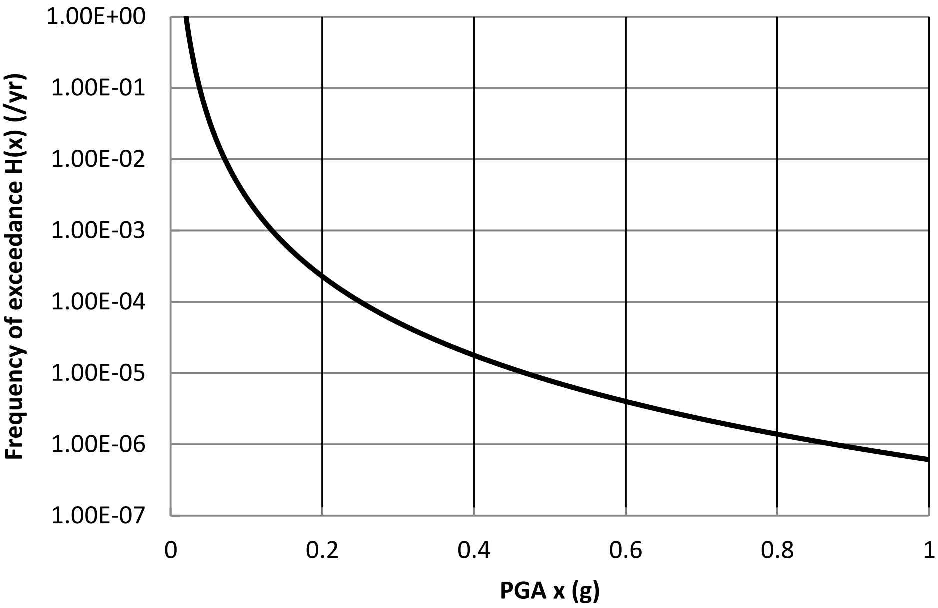

Figure 1: Seismic hazard – Power Law model normalised to $H(0.25\text{g}) = 10^{-4}$/yr

Fragility Models: The standard separation of variables type lognormal fragility model commonly used for seismic analysis is assumed here (see e.g. Kennedy 2000); the traditional two parameter uncertainty model is replaced by the equivalent one-parameter model, again for simplicity. In density and cumulative form this can be written as

$$f(x) = \frac{1}{\sqrt{2\pi}x\beta}\cdot \exp\left[-\frac{\ln^2(x/\breve{x})}{2\beta^2}\right] \quad \text{and} \quad F(x) = \Phi\left[\frac{\ln(x/\breve{x})}{\beta}\right] \quad (6)$$

where Φ is the standard cumulative normal distribution, $\breve{x}$ is the median PGA value and β is the lognormal standard deviation of the fragility curve.

We further assume that the design bases of interest are such that a well-designed plant will have only a 1% chance of failure at hazard values up to the design basis x_{DB} and a 99% chance of failure beyond this. This point on the fragility model is equivalent to the well-known High Confidence Low Probability of Failure (HCLPF) point on the two-parameter uncertainty model of fragility. For the one-parameter model, the hazard value at which this occurs is

$$x_{1\%} = \breve{x} \cdot e^{-2.33\beta} \quad (7)$$

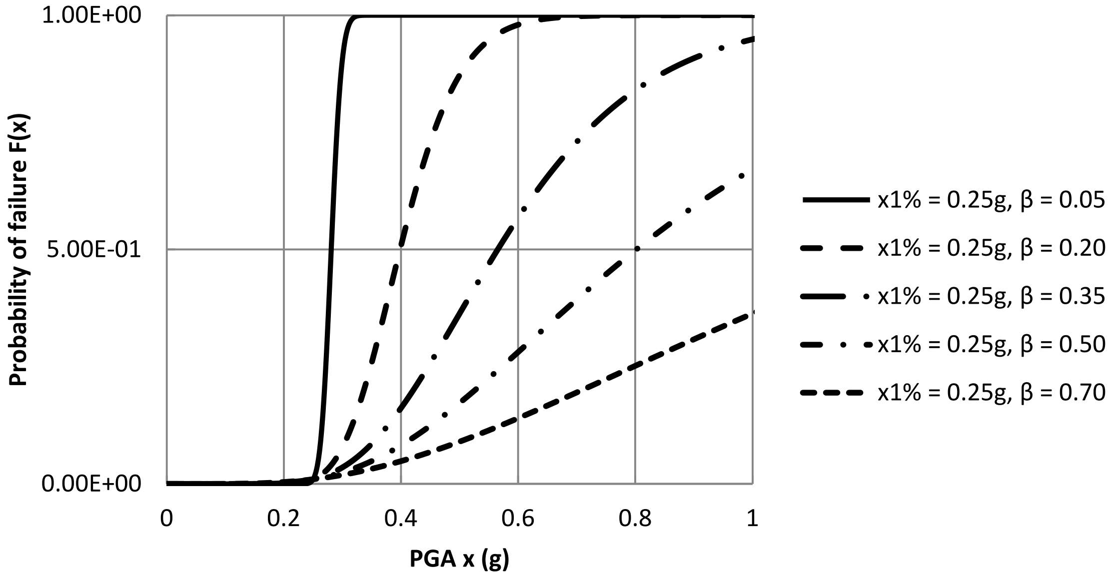

Figure 2a: Fragility curves for lines of protection with parameters defined in Table 3

Figure 2a illustrates a number of fragility curves with parameters as listed in the key; the fragility density curves themselves are shown in fig. 2b.

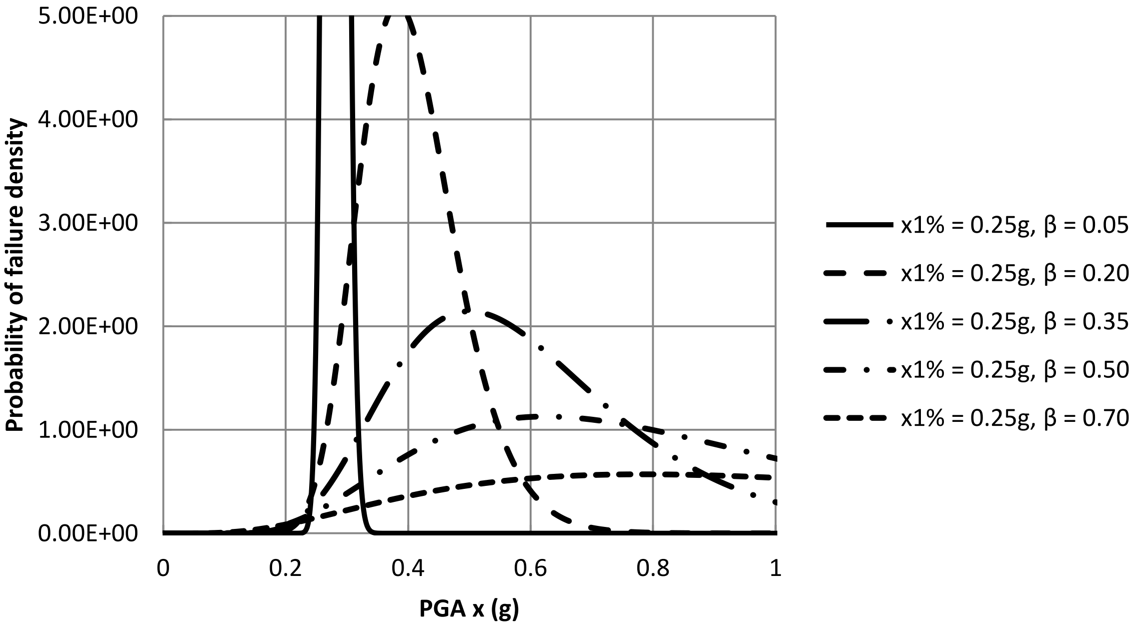

Figure 2b: Probability density functions for fragility curves in Figure 2a.

Frequency of failure under seismic hazard loads: Using the power law hazard model and the one-parameter fragility model, Kennedy developed a solution to eqn. (3) as a frequency of failure relationship for a single line of protection:

$$P(\infty) = h_0 \breve{x}^{-n} \cdot e^{n^2\beta^2/2} \quad (8)$$

We extend eqn. (8) here to a solution for two ANDed lines of protection, i.e. a solution to eqn. (4):

$$\mathrm{P}_{A\cap B}(\infty) = P_A(\infty) \cdot \Phi\left[\frac{\ln(\breve{x}_A/\breve{x}_B) - n\beta_A^2}{(\beta_A^2+\beta_B^2)^{1/2}}\right] + P_B(\infty) \cdot \Phi\left[\frac{\ln(\breve{x}_B/\breve{x}_A) - n\beta_B^2}{(\beta_A^2+\beta_B^2)^{1/2}}\right] \quad (9)$$

Application to Seismic Risk, Risk Targets: In the UK, a number of risk targets are defined in the ONR's Safety Assessment Principles (ONR 2014). Any real nuclear licensed site in the UK has to satisfy all of these to the extent required by the principle of keeping risks As Low As Reasonably Practicable (ALARP), which is the overriding legal requirement in the UK. For the purpose of illustration, we select a Target 8 release, which is relevant generally to large civil reactor plants and relates to a large release where radiation doses to a member of the public could be greater than 1000mSv. Total predicted frequencies of accidents should not exceed a *Basic Safety Limit* of 10^{-4}/yr and operators should aim for a lower value called a *Basic Safety Objective* of 10^{-6}/yr. Moreover, no single class of accident should make a disproportionate contribution to these values; so for the purposes of this analysis, since seismic hazard forms a single class, these values are reduced by an order of magnitude to 10^{-5}/yr and 10^{-7}/yr. These set benchmarks against which the frequencies of plant failure computed from eqns. (8) & (9) can be compared.

Seismic risk for plants protected by one or two lines of protection: The beneficial effect of adding a second line of protection to the first line is easily demonstrated by applying eqns. (8) & (9) to a simple representative plant. Assume as follows:

Table 1: Fragility parameters for lines of protection *A* and *B*

Line of protection	$x_{1\%} = x_{DB}$	β	$\breve{x}$	$H(x_{1\%})$
A	0.25g	0.35	0.565	10^{-4}/yr
B	0.125g	0.35	0.283	1.3 x 10^{-3}/yr

Then consider the four situations:
- Line A exists in isolation.
- Line B exists in isolation.
- The ANDed combination A and A $(A \cap A)$ exists[6].
- The ANDed combination A and B $(A \cap B)$ exists.

The risk arising from a seismically initiated fault protected in this way, expressed in frequency of failure format, $P(\infty)$, is computed from eqns. (8) and (9) as:

Table 2: Frequency of failure for plant with lines of protection defined in Table 1

$P_A(\infty)$	$P_B(\infty)$	$P_{A\cap A}(\infty)$	$P_{A\cap B}(\infty)$
1.1 x 10^{-5}/yr	1.5 x 10^{-4}/yr	4.1 x 10^{-6}/yr	9.4 x 10^{-6}/yr

[6] These LoP do not have to be physically identical, but are assumed to have identical fragility parameters and be seismically independent and uncorrelated.

The graphs below show frequency of failure in both density and cumulative form for all these examples. To do this eqns. (3) and (4) were solved for $P(X), (0 < x \leq X)$, using a first order finite difference algorithm, and the density forms, $p(X)$, of these equations were solved in similar fashion. The step length, ΔX, of the algorithm was adjusted so as to compute the $P(\infty)$ values in Table 2 to two significant figures:

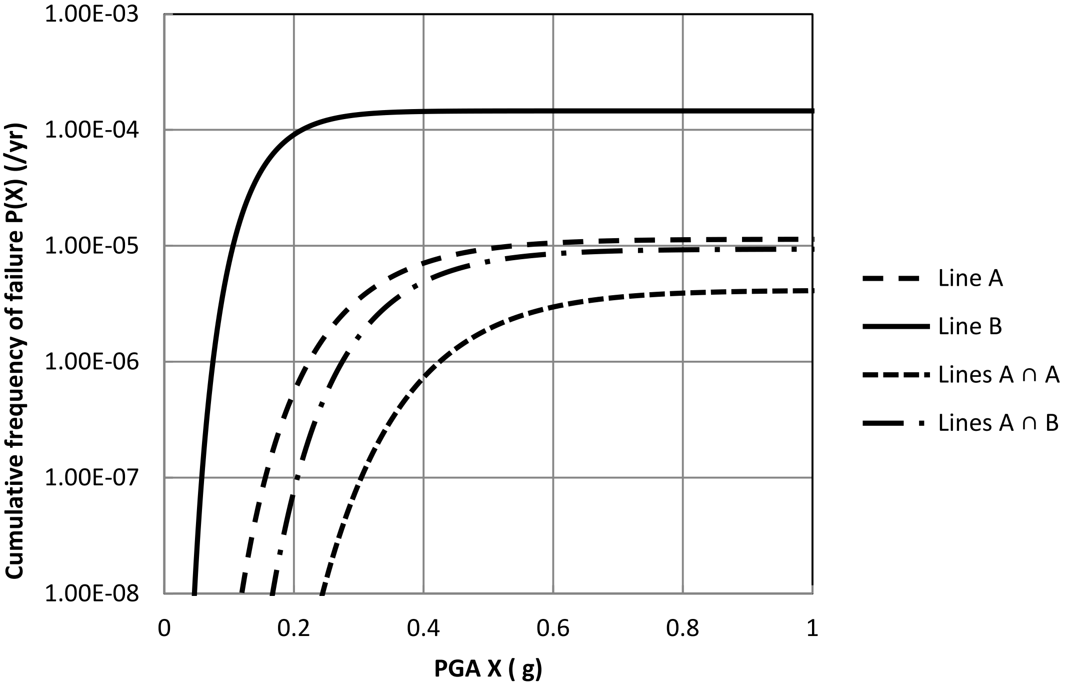

Figure 3: Cumulative frequency of failure function, $P(X)$, for four different combinations of lines of protection. Note $P(\infty)$ is reached well before $X = 1g$ in all cases.

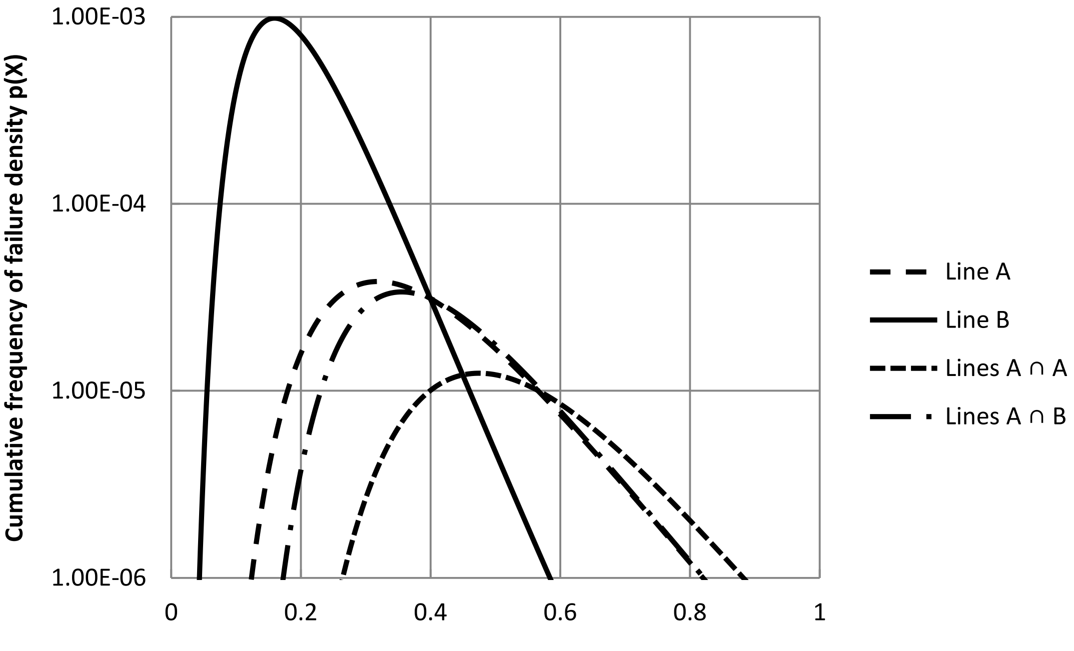

Figure 4: Frequency of failure density function, $p(X)$, for the cumulative curves in Figure 3

Deductions
Several things are apparent from Figures 3 & 4:

- Firstly, Figure 3 clearly shows that for A alone and for combinations of A and B, most of the contribution to failure occurs just beyond the design basis value of 0.25g, up to about twice this value, 0.5g. We assume this result is typical, although the effect will depend on the fragility and hazard parameters selected, and the LoP combinations being considered. It illustrates the importance of considering beyond design basis plant performance and the nature of hazard induced fault progression, because it is in this region that the plant is most likely to fail. Note that the peak in the density functions in Figure 4 occur in this region.

- A second feature is the degree of benefit to be gained by adding a second LoP to the first. Note however that this only applies when the two lines are of similar strength, see comparison of A and $(A \cap A)$. When A is combined with B, the frequency of failure hardly improves at all. This illustrates that overall, plant failure frequency is driven primarily by the quality of the LoP with the higher design basis, A in this case. Additional lines, with design bases much below this, are of limited value for controlling overall plant risk.

- Third, a very important feature of seismic hazard is that it has a major common cause effect on all the plant's lines of protection. Combinations of LoP reduce overall plant failure frequency, but by much less than would be the case if the failure of each line were initiated by independent random events. In this example, the combination $A \cap A$, while useful, provides less than an order of magnitude improvement in failure frequency over A alone, whereas if the initiating events had been truly independent the failure frequency would be: $P_{A\cap B}(\infty) = P_A(\infty) \times P_A(\infty) \sim 10^{-10}$ /yr, a rather better figure.

- A fourth important feature relates to the fact the hazard curve is expressed as an exceedance frequency, which is used here as a surrogate for exceedance probability. Thus, at any selected value of the hazard function, say $H(0.25\text{g}) = 10^{-4}$/yr, there is a 10^{-4}/yr chance (frequency) that an earthquake will occur of severity in the range $x \geq 0.25$g. This is why, even with a 1% plant failure probability at the design basis hazard level, overall frequency of failure for a single line of protection can never approach $10^{-4} \times 10^{-2}$/yr $= 10^{-6}$/yr!

THE EFFECT OF SEISMIC MARGINS
Our measure of margin is defined by eqn. (1). When applied with eqn. (7) to this example, it is easy to show that the seismic margin is $\Delta = e^{-2.33\beta}$. We have selected five values of log-standard deviation, β, and the details for each LoP are shown below.

Table 3: Fragility parameters for lines of protection for Table 4 and Figure 5

Case	Line A parameters				Line B parameters			
	$a_{1\%}$ (g)	β	$\breve{a}$ (g)	Δ	$a_{1\%}$ (g)	β	$\breve{a}$ (g)	Δ
1	0.25g	0.05	0.28	1.12	0.125g	0.05	0.14	1.12
2	0.25g	0.20	0.340	1.59	0.125g	0.20	0.20	1.59
3	0.25g	0.35	0.57	2.26	0.125g	0.35	0.28	2.26
4	0.25g	0.50	0.80	3.21	0.125g	0.50	0.40	3.21
5	0.25g	0.70	1.28	5.11	0.125g	0.70	0.64	5.11

We present results for each case, in Table 4 and Figure 5, to illustrate the effect of varying the margin, Δ, and have assumed for convenience that for combinations of A and B both lines have the same value of Δ.

Table 4: Frequency of failure for plant with lines of protection defined in Table 3

Case	Δ	$P(\infty)$ (/yr)			
		A	B	$A \cap A$	$A \cap B$
1	1.12	6.6 x 10^{-5}	8.5 x 10^{-4}	5.9 x 10^{-5}	6.6 x 10^{-5}
2	1.59	2.4 x 10^{-5}	3.0 x 10^{-4}	1.4 x 10^{-5}	2.3 x 10^{-5}
3	2.26	1.1 x 10^{-5}	1.5 x 10^{-4}	4.1 x 10^{-6}	9.4 x 10^{-6}
4	3.21	7.5 x 10^{-6}	9.6 x 10^{-5}	1.5 x 10^{-6}	3.9 x 10^{-6}
5	5.11	6.8 x 10^{-6}	8.7 x 10^{-5}	4.7 x 10^{-7}	1.4 x 10^{-6}

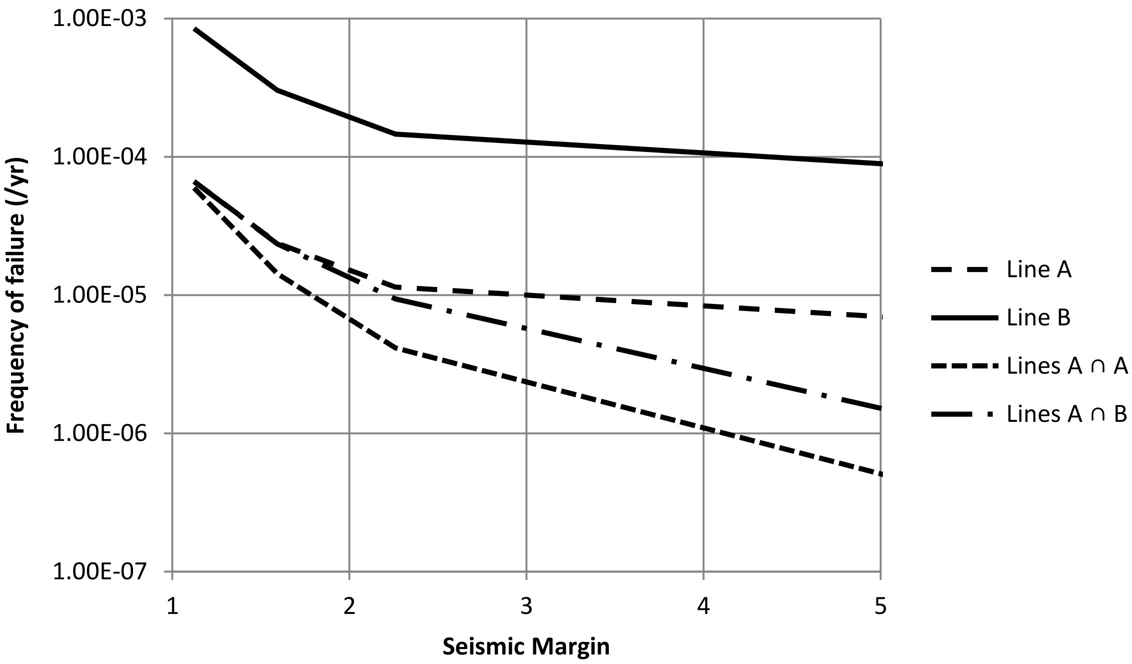

Figure 5: Frequency of plant failure v. seismic margin for lines of protection presented in Table 4

Deductions

Again, several points are worthy of note from this graph:

- As seismic margin tends to unity, failure frequency tends to the design basis frequency: As $\Delta \rightarrow 1$, then $P(\infty) \rightarrow H(a_{1\%}) = H(a_{DB})$.

- For single lines of protection, while frequency of plant failure decreases with increasing seismic margin, $\Delta > 3$ offers little additional benefit, although this depends on the hazard and fragility parameters chosen. Also, single lines offer about a factor of 10 improvement in failure frequency over the design basis hazard frequency at $\Delta > 3$; so to achieve BSO levels of ~10^{-7}/yr would require design bases at the 10^{-6}/yr hazard frequency, or about three times higher in terms of PGA for the example used in this paper, i.e. $x_{DB} = x_{1\%} \approx 0.75$g.

- Employing two lines of protection offers significant advantage, about a factor of 10 or more in plant failure frequency if both lines have the same design basis, but much less if the second line has a significantly lower design basis. Also, this advantage does not plateau off in the same way beyond $\Delta > 3$, although the curves for two lines of protection do shallow significantly beyond this value.

DISCUSSION

The seismic risk calculations presented here refer to a "high seismic" nuclear site in the UK and the plant model is intended to represent a grossly simplified version of a commercial nuclear reactor. Nevertheless, the calculations illustrate how the application of numerical seismic risk analysis can be used to demonstrate compliance with numerical risk criteria. The important points made in this paper are:

- Seismic hazard contributes to plant risk across a wide range of hazard frequencies, not just at the 10^{-4}/yr design basis level. The contribution to plant risk is most pronounced in the region just beyond the design basis value, up to about twice this value. This suggests that the analysis of beyond design basis plant response is important to demonstrate plant safety is adequate and risk is ALARP. This has been reflected in the increased emphasis on BDBA in the most recent edition of ONR's SAPs.

- Multiple independent Lines of Protection are a valuable safety feature, but only if they are truly independent in terms of their seismic response, i.e. their seismic response is independent and uncorrelated, and they are designed to similar design basis values.

- The "probability of exceedance hazard curve" nature of seismic events means that hazard severity is always likely to overtake plant withstand capability at very low probabilities (frequencies) of exceedance. Based on the simple plant models used here, this is likely at exceedance frequencies of order 10^{-6}/yr, or just those target frequencies forming BSO criteria used in the UK.

CONCLUSIONS

This paper presents an application of a simplified seismic risk analysis to a notional nuclear plant to demonstrate how such analysis can be used to examine compliance with risk criteria, and how design basis and beyond design basis margins can influence seismic risk. The plant model accommodates both single and two Lines of Protection and results are provided to compare the risk implications in both cases. Results are also presented to demonstrate the importance of characterising seismic hazard by a full exceedance frequency hazard curve, rather than just considering the design basis value in isolation. The implications of this on seismic risk are illustrated by worked examples. This analysis has informed the additional contribution on BDBA in the recent edition of ONR's SAPs.

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APPENDIX – DERIVATION OF FREQUENCY OF FAILURE FOR TWO ANDED COMPONENTS

For the AND combination of LoP, failure frequency is described by eqn. (9). To derive this the starting point is the integral from eqn. (3) in the form:

$$P_{A\cap B}(\infty) = \int_0^\infty H(x) \cdot [f_A(x)F_B(x) + F_A(x)f_B(x)] \cdot dx \tag{10}$$

With *H*(*x*) given by the power-law model eqn. (8), and the fragility curves by the lognormal expression eqn. (6), this integral can be evaluated analytically as follows. The first of the two terms can be written out as

$$I_1 = \frac{h_0}{\sqrt{2\pi}\beta_A}\int_0^\infty x^{-n-1}\exp\left[-\tfrac{1}{2}\ln^2\left(x/\breve{x}_A\right)/\beta_A^2\right]\Phi\left[\beta_B^{-1}\ln\left(x/\breve{x}_B\right)\right]dx \tag{11}$$

With the substitution $y = [\ln(x/\breve{x}_A) + n\beta_A^2]/\beta_A$ this becomes

$$I_1 = \frac{h_0}{\sqrt{2\pi}}\breve{x}_A^{-n}\mathrm{e}^{\frac{1}{2}n^2\beta_A^2}\int_{-\infty}^\infty \mathrm{e}^{-\frac{1}{2}y^2}\Phi\left[\frac{\beta_A y - n\beta_A^2 + \ln(\breve{x}_A/\breve{x}_B)}{\beta_B}\right]dy \tag{12}$$

Now the integral can be evaluated by observing that

$$I_2 = \frac{1}{2\pi}\int_{-\infty}^\infty \mathrm{e}^{-\frac{1}{2}y^2}\Phi(py+q)dy = \frac{1}{2\pi}\int_{-\infty}^\infty\int_{-\infty}^{px+q}\mathrm{e}^{-\frac{1}{2}(y^2+z^2)}dzdy \tag{13}$$

This is the integral of the circular normal distribution over the plane under the line z = *py+q.* By rotating the axes so that this line becomes parallel to the y-axis at a distance $q/\sqrt{1+p^2}$ to separate the variables, one obtains the result

$$I_2 = \Phi\left(q/\sqrt{1+p^2}\right) \tag{14}$$

Thus we get

$$I_1 = h_0\breve{x}_A^{-n}\mathrm{e}^{\frac{1}{2}n^2\beta_A^2}\Phi\left[\frac{\ln(\breve{x}_A/\breve{x}_B) - n\beta_A^2}{\sqrt{\beta_A^2+\beta_B^2}}\right] \tag{15}$$

which is the first term in eqn. (9). The second is obtained by interchanging *A* and *B*.