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STRUCTURE SOIL STRUCTURE INTERACTION UNDER SEISMIC CONDITIONS, WITHIN THE UK CONTEXT

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Abstract: *When constructing a new nuclear power in the UK, a robust level of resilience is expected as reflected in the strict UK regulatory requirements. As this takes place post-Fukushima and several other significant nuclear power plant accidents around the world, the expectations in the design of the structures to withstand adversity are heightened. This includes extreme events such as a 1 in 10,000-year earthquake as well as events beyond that. Therefore, to ensure an adequate level of resilience, all plausible eventualities must be considered in the design. This paper discusses the civil design performance requirements for a class 1 nuclear safety related structure. As there are several structures on site and following International Atomic Energy Agency (IAEA) guidance that structure soil-structure interaction effects need to be considered for a new nuclear facility, both soil-structure interaction (SSI) as well as structure-soil-structure interaction (SSSI) analysis are studied as part of the generic design process of buildings. This paper introduces both concepts of soil-structure interaction (SSI) and structure-soil-structure interaction (SSSI) and presents the use of LS-DYNA as a 3-D finite element software with the capability to simulate dynamic SSI and SSSI effects using explicit time integration. It also discusses the theoretical principles to perform this analysis, as well as inputs, outputs and limitations of the program.*

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Introduction

Project Background

A number of new nuclear power stations are planned in the UK to provide a stable source of electricity for the general population. The process of designing such structure involves analysis and design including all different aspects, ranging from static conditions to extreme conditions, like seismic.

All the structures are generally analysed and designed in isolation, but the requirements of assessing the potential relevance of the effects of structure–soil–structure interaction is taken into account in an explicit analysis described herein.

The main reactor building generally consists of a number of sub-structures founded together or in a close proximity. As a result, these structures combined have a significant mass with the potential to affect the ground response of other structures on site. There is no specific requirement related to this effect, but relevant good practice requires consideration of such effects, for example International Atomic Energy Agency (IAEA) SSG-67 (2021).

Furthermore, the Office for Nuclear Regulation (ONR) in the UK, requires relevant good practice to be incorporated in the design process. Therefore, projects can employ a class leading structure-soil-structure interaction (SSSI) analysis to establish any impact on the design aspects from such effect, through the use of aggravation factors.

Therefore, this paper focuses on the LS-DYNA SSI analysis of two structures, a main building (referred to as M building) and a separate building (referred to as S building), which are adjacent to each other. This work is following a benchmark study and detailed soil-structure interaction analysis of these buildings, when considered separately, which is not presented in this paper.

The product of these efforts is the production of the aggravation factors, which indicate the severity of SSSI effect, and associated recommendations for the detailed design stage of the buildings.

Soil-Structure Interaction (SSI) And Structure-Soil-Structure Interaction (SSSI)

Earthquake waves originate from a fault rupture and propagate in the earth's crust. In the absence of structures, the resulting ground motion is called the free-field ground motion. When a structure is present, the ground motion results in structural vibration.

Large civil structures such as concrete dams, nuclear power plants and bridges are massive enough that their vibration due to earthquake excitation affects the motion of the soil supporting them, which in turn further affects the motion of the structure itself.

According to IAEA (2021) structure–soil–structure interaction (SSSI) refers to a phenomenon by which the seismically induced motion of a structure is transmitted to an adjacent structure through the foundation medium. A typical effect of this phenomenon is that, in the in-structure spectra of the affected structure, peaks appear at the natural frequencies of the adjacent structure. When structure–soil–structure effects are deemed to be potentially relevant, they should be considered in the design, particularly for the development of in-structure response spectra to be used for qualification of systems and components housed by the main structures.

There are a number of software packages available on the market that can be used to perform SSI and SSSI studies, such as FLUSH and SuperFLUSH for 2-dimensional analysis. CLASSI, SASSI, FLUSH3D and LS-DYNA for 3-dimensional analysis. This paper presents the use of LS-DYNA for SSI and SSSI, but it is worth noting that the project did use SuperFLUSH in the preliminary stages, which is not covered by this paper.

LS-DYNA as an SSI tool

LS-DYNA is a 3-D finite element software with the capability to simulate a number of different structural and mechanical aspects. One of them relates to dynamic soil-structure interaction (SSI) and structure-soil-structure interaction (SSSI) analysis problems.

Both explicit and implicit integration algorithms are implemented in LS-DYNA, with the implicit formulation having limited capabilities. However, it does allow modal analysis of structures, which helps the verification efforts.

Application of LS-DYNA to SSI analysis is documented for a variety of civil and structural engineering applications as detailed in Hallquist, J., (2006) and Bolisetti and Whittaker (2015).

Within the LS-DYNA software there are two variations of solver available. The first is a single precision solver and the second is a double precision solver. The results can vary dramatically depending on which solver is used and the level of accuracy required. The double precision solver can increase run times by up to 30%, but may be necessary to use for analysis requiring high accuracy. This is generally the case for analysis with long durations and relatively low displacements. Using the single precision solver (8 decimal places) may result in an error in the displacements at the end of the analysis.

The key capabilities in LS-DYNA that are relevant to the requirements of such SSSI studies include:

- The seismic free-field ground motion at the soil-structure interface which can be incorporated using either; effective seismic input method - relating to a scattering analysis framework as developed by Bielak et al, or a direct method – where the seismic input is applied at the bottom of the soil domain.
- An unbounded domain which is simulated using Perfectly Matched Layers (PML), a type of absorbing boundary.
- The structures can be idealised using Belytschko-Schwer beam elements.
- LS-DYNA code allows for both explicit and implicit solutions, hence a fixed-base, modal analysis may be performed for the modelled structure to obtain modal frequencies, mass-participations and mode shapes.

Theoretical principles

The basis for the SSI and SSSI analyses in LS-DYNA that were considered for the studies, relates to the effective seismic input method and the use of absorbing boundaries. These principles are provided in the following subsection along with additional details of the explicit time algorithm and modelling of structural damping and mass-scaling solution approach.

Effective seismic method

Within the context of the discussed studies, SSI analysis was undertaken in LS-DYNA using the effective seismic input method. This method is based on an approach where the loads imparted to a structure during an earthquake are determined by considering the effect of scattering incoming waves. This approach was developed by Bielak et al. and is described by Herrera and Bielak (1977) and Bielak and Christiano (1984) and is incorporated into LS-DYNA.

The scattering analysis approach considers soil-structure interaction to be caused by the scattering of the free-field ground motion by the presence of a structure.

Firstly, considering the scenario where there is no structure, as shown in Figure 1 a). This scenario represents the free field condition which is the condition that the earthquake loading is specified for and is the basis for the structural calculations.

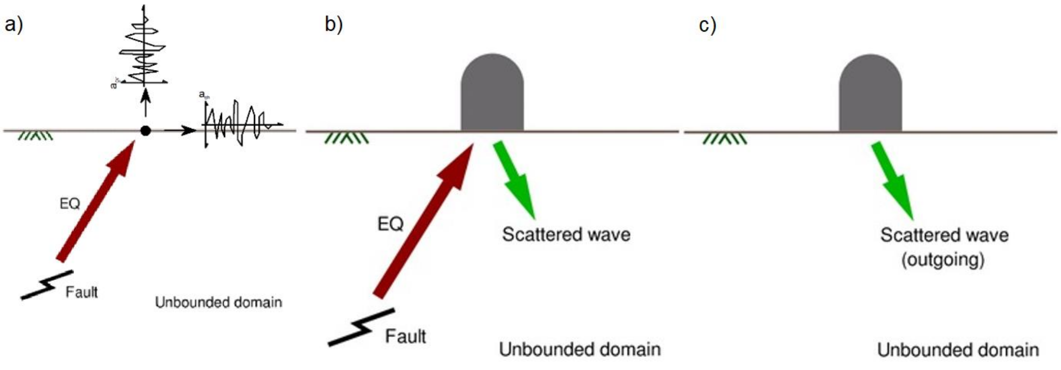


Figure 1. Effective seismic method – scattered waves

Following on and considering the scenario where a structure is placed within the domain is presented in Figure 1 b). In this scenario, the presence of the structure causes the incoming waves to be scattered which produces reflections back into the soil domain.

If incoming motions between the two scenarios are subtracted from each other the resulting motion is the scattered motion, see Figure 1 c). This scatter motion is directly related to the seismic forces at the interface between the structure and the soil.

The unbounded domain can be replaced by a bounded domain, surrounded by a numerical formulation of an absorbing boundary layer, which helps reduce the wave reflections back to the model, as shown in Figure 2.

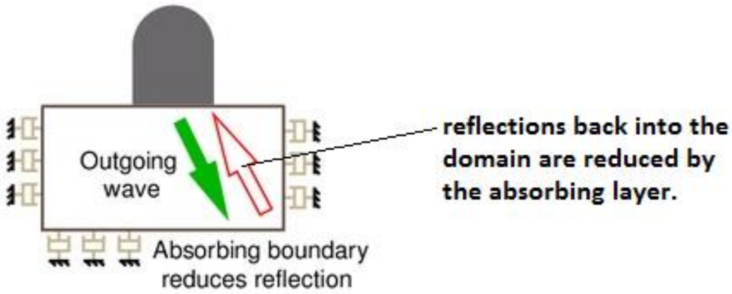
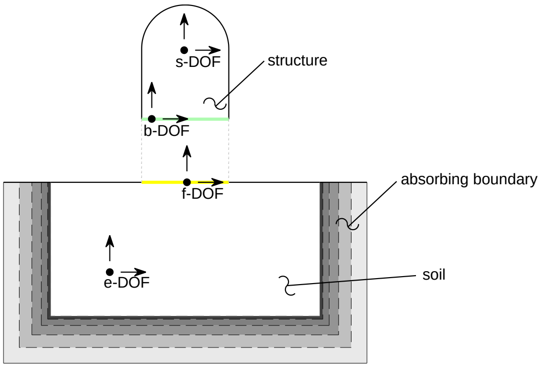


Figure 2. Bounded domain

Within LS-DYNA the absorbing boundary is provided by a feature defined as a perfectly matched layers (PML), and the scattered-wave formulation will provide the equivalent earthquake forces, termed as the effective seismic input. The idealisation shown in Figure 2 is repeated in greater detail in Figure 3 along with the equation which describes the system.



$$\begin{bmatrix} M_{ss} & M_{sb} & 0 \\ M_{bs} & (M_{bb}+M_{ff}) & M_{fe} \\ 0 & M_{ef} & M_{ee} \end{bmatrix} \begin{Bmatrix} \ddot{u}_s \\ \ddot{u}_b \\ \ddot{u}_e \end{Bmatrix} + \begin{bmatrix} K_{ss} & K_{sb} & 0 \\ K_{bs} & (K_{bb}+K_{ff}) & K_{fe} \\ 0 & K_{ef} & K_{ee} \end{bmatrix} \begin{Bmatrix} u_s \\ u_b \\ u_e \end{Bmatrix} = \begin{bmatrix} 0 \\ M_{ff} \\ M_{ef} \end{bmatrix} \ddot{u}_f^0 + \begin{bmatrix} 0 \\ K_{ff} \\ K_{ef} \end{bmatrix} u_f^0$$

Figure 3. Typical SSI system within LS-DYNA after Hallquist (2006)

Note: As shown in Figure 3, the matrices M_{ij} and K_{ij}, are submatrices of the mass and stiffness matrices of the structure (i, j = s, b) and of the soil (i, j = f, e), which include explicitly the absorbing boundaries. Note that u_s and u_b denote total nodal displacements within the structure, and on the interface between the structure and the soil, on the structure side. Within the soil, however, u_e denotes scattered motion, i.e. the relative displacement with respect to the free-field motion, as described above.

The formulation of the system shown in Figure 3 is presented by Bielak and Christiano (1984) for continuum and discretised problems. The version presented here is a simplified linear version which is incorporated into the LS-DYNA theory documentation Hallquist (2006).

The left-hand side of the equation contains known properties and the response parameters of the system. The right-hand side of the equation contains the loading which is commonly referred to as the effective seismic forces.

For a discretised model the accuracy depends solely on the ability of the artificial boundaries, which simulate an infinite domain, to absorb the scattered wave. For the purposes of the effective seismic method, artificial boundaries are employed within the model. These consist of an absorbing boundary called the PML.

PML absorbing boundaries

The PML implemented in LS-DYNA was originally developed by Basu and Chopra (2003, 2004) and Basu (2009). When placed next to an elastic bounded domain, the PML absorbs and attenuates nearly all the outward travelling waves from the bounded domain, minimising reflection from the interface between the bounded domain and the PML as shown in Figure 4.

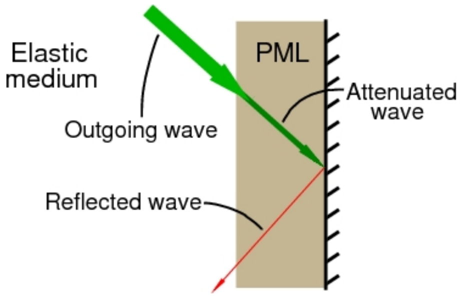


Figure 4. Perfectly matched layer (PML) boundary

The PML is explicitly modelled and absorbs waves from the elastic bounded domain that it is placed next to and is composed of the same material of the adjacent bounded domain. The PML typically requires between 5 to 8 elements through the depth, with the outer boundary fixed in all degrees of freedom after LSTC (2017).

Other considerations

LS-DYNA explicit solver is used to determine the SSSI effects, which does not require iteration to achieve equilibrium at each time step and it does not require the inversion of the stiffness matrix, thus making the explicit analysis less computationally expensive. The downside to these benefits is that the required time-step for explicit analysis is small and it may be orders of magnitude lower than implicit analysis time-steps. This results in long run times, but the effective seismic input method implemented into the explicit solver is the solution technique that is used in the presented studies.

There are a number of available damping methods within LS-DYNA. A frequency-independent damping method provides constant damping over the whole frequency range, but this type of damping is only intended for small damping ratios (e.g. <0.05), with the drawback that there is an impact on the dynamic stiffness of the model. As such, LS-DYNA recommends that the elastic stiffness in the model be increased or decreased depending on the option selected to account for the damping across a frequency range. The estimated frequency errors for damping ratios <4% are presented in the ‘LS-DYNA Manual Volume I’ along with the corresponding percentages recommended by LS-DYNA to increase or decrease the elastic stiffness in the model. The frequency-independent damping is used for the analysis presented in this paper, to capture the soil related damping, but as it is applied to the whole model, the stiffness adjustment is considered in line with the LS-DYNA manual, corresponding to 4% damping level, for every element.

This damping method is implemented in the studies presented in this paper, however, as it is only recommended for small damping ratios an additional damping method is also considered, using the Rayleigh viscous damping approach. The Rayleigh method is proportional to a linear combination of mass and stiffness and supplements specific damping requirements for buildings, i.e., over and above general constant damping for the whole model. For majority of buildings the total damping level of 7% is applied, in line with ASCE 4-16 requirements, for reinforced concrete structures, resulting in 3% additional Rayleigh damping for structures.

The solution time step for an explicit analysis is determined internally by LS-DYNA and is based on the speed of propagation of stress waves within the structure. A technique commonly employed in explicit analysis is to adjust the density of small elements which control the time step to reduce the time taken for stress waves to propagate through them. This technique is known as mass scaling.

Mass scaling is an automated procedure whereby LS-DYNA increases the time step by scaling up the density in the specific elements that are controlling the time step. Based on the duration requirements, a minimum time step size is manually specified in the LS-DYNA command that controls the timestep. To determine the amount of mass scaling required to reduce the duration of the analyses without impacting the results a sensitivity study was conducted, using various percentages of mass scaling, e.g., 0%, 0.3%, 2%, 5%, 10% and 40%. The response spectrum results at the top of a stick in the vertical direction is presented in Figure 5.

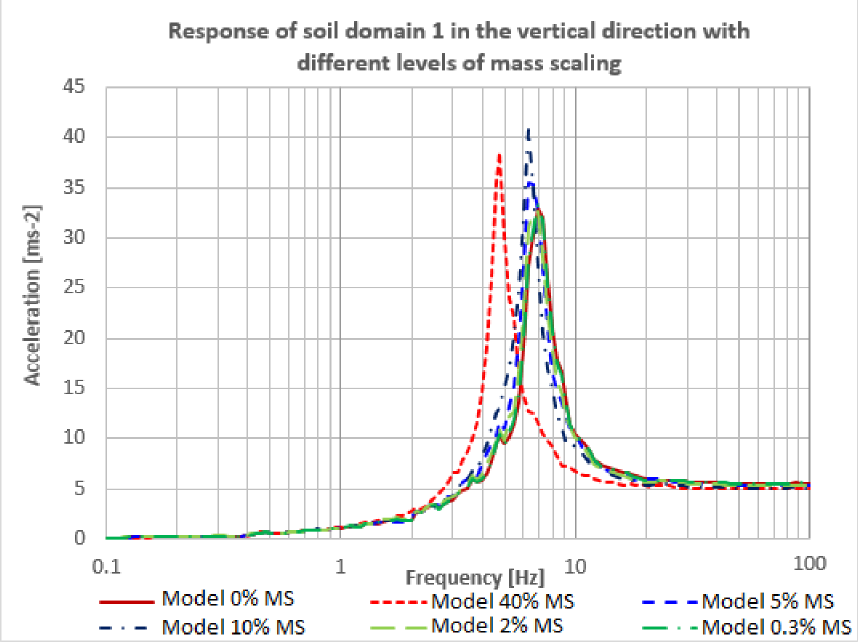


Figure 5. Response spectrum at the top of a stick showing the effect of mass scaling

As can be seen from Figure 5, when mass scaling is higher than 10% it can have a significant effect on the global response of the system. A similar comparison was carried out at the bottom of the stick for the horizontal direction which indicated that mass scaling of 5% had a very small impact and 2% was almost negligible. Therefore, it was concluded that although the appropriate degree of mass-scaling is determined on a case-by-case basis, if generally kept <5% of the total structural mass this will not significantly affect the response of the system. As a result, the main analysis runs used 3% of mass scaling, with no impact on the solution.

Inputs

The input to LS-DYNA consists of a series of keywords identifying the specific processes and commands to be implemented when solving the analysis, with the primary keywords summarised in the appendix of this paper.

Outputs

Time histories of accelerations, velocities, displacements, forces, stresses and strains are the primary outputs from LS-DYNA. The acceleration time histories were extracted from the analysis models and converted to secondary response spectra.

Aggravation factors

As discussed above, the primary outputs from the analysis are time histories of accelerations, which are converted using a single degree of freedom approach into secondary response spectra. The aim of the study is to establish the SSSI effects by comparing the SSI analysis results from a single building with the SSSI analysis results from two buildings. A graphical representation of such comparison at the base of the structure is presented in Figure 6, in idealised situation, where two different spectra are directly compared on a single graph, indicating the severity of the multi structure interaction effects.

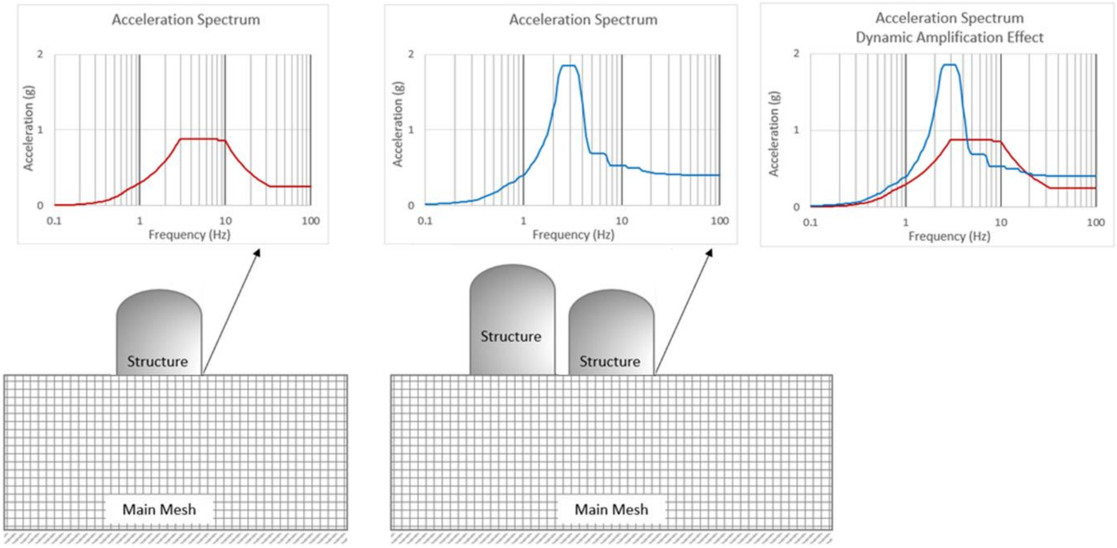


Figure 6. Definition of the Aggravation factor representing the dynamic SSSI amplification effects

To numerically establish the aggravation function, the two spectra are divided by each other in the frequency domain as presented in equation 1. The final aggravation factor is established as a maximum value in the whole frequency range as presented in equation 2.

$$Aggravation\ function(freq) = \frac{SRS_{SSSI}(freq)}{SRS_{SSI}(freq)} \qquad \text{(Equ.1)}$$

$$Aggravation\ factor = Max(Aggravation\ function(freq)) \qquad \text{(Equ.2)}$$

The aggravation factors are to be directly used in the design process, by applying them to the design spectra used in the response spectrum analysis or directly to time histories if full time history analysis is employed.

Limitations of the LS-DYNA software package

The following limitations are associated with using LS-DYNA explicit time integration to derive the SSSI response:

- Beam modelling – LS-DYNA offers a number of methods to input the sectional properties of a beam, as well as different modelling approaches. However, for a beam with user defined sectional properties, LS-DYNA does not allow two different shear areas to be input. This limits the flexibility of beam modelling and ultimately may require some property modification to be carried out on the beams, i.e. LS-DYNA equivalent beam properties.
- Time step – In explicit analysis the time step is calculated internally to provide a stable solution. This can quite often result in analysis with long runtimes. This can be mitigated using mass scaling; however, only very small amounts of mass should be added to areas where critical results are being derived.
- Rigid massless elements – The time-step of an explicit analysis solver is dependent on the modulus of elasticity and the density of the elements. Therefore, zero mass elements result in very small time-steps and should be avoided if possible.

Modelling

The general schematics of SSI approach in the LS-DYNA software package are summarised graphically in Figure 7. The boundaries of the soil domain are modelled using the absorbing PML boundary, as indicated in the figure. Using an absorbing boundary means that a smaller soil domain can be used without compromising accuracy. A smaller soil model in turn reduces the overall computational time. Fixed boundary conditions are applied to the external face of the PML, including the base.

The buildings are simplified into stick models, with multiple degrees of freedom, capturing the global behaviour of the actual structures. In LS-DYNA, the stick and explicit slab are combined with the soil model via a tied-contact soil-structure interface where the ground motion is applied in three directions, using the effective seismic input method

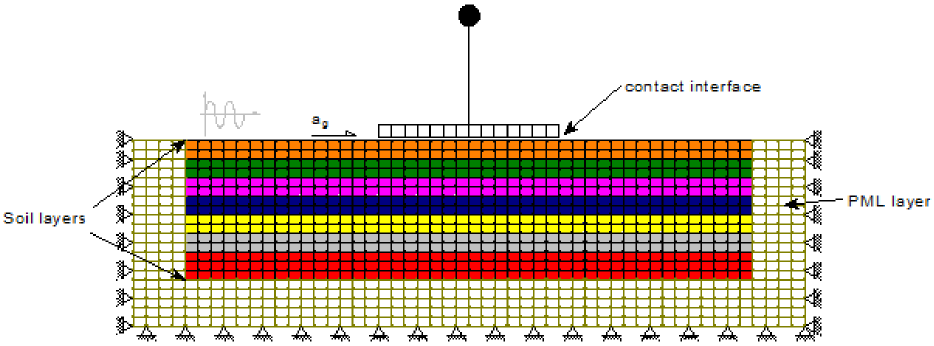


Figure 7. Schematics of SSI system used in LS-DYNA

Generally, all structures are subject to 7% of damping in the LS-DYNA analysis. The damping method that is used in the LS-DYNA model is a combination of frequency range (general over the entire model of 4% i.e., including the soil model) and Rayleigh damping (remaining 3% - structures only).

A screenshot of a simplified single building SSI model in LS-DYNA, representing simplified S building with a single degree of freedom stick can be seen in Figure 8. Due to a relatively small soil domain, the PML layers are significantly extended, which improves the speed of the analysis.

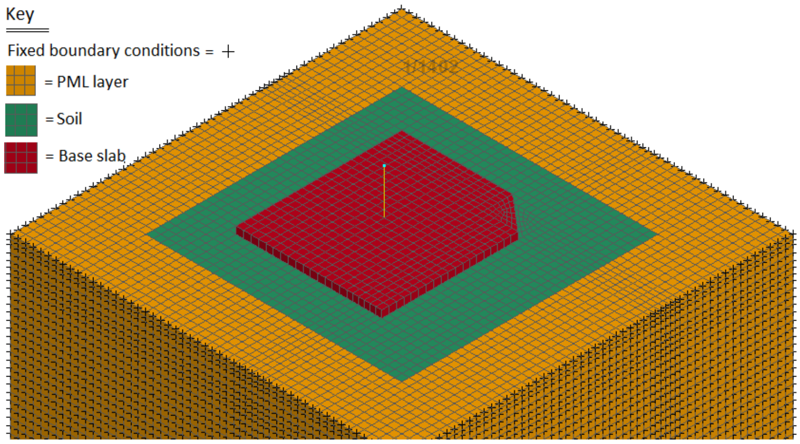


Figure 8. LS-DYNA finite element model showing PML and boundary conditions

Note: there is no clear guidance onto what the requirements are for the minimum size of the soil domain. In order to investigate the sensitivity of the size of the soil domain, 4 variations of soil domain sizes were analysed (50m x 50m; 70m x 70m; 120m x 120m & 160m x 160m). Variations of mass scaling and mesh refinement were also investigated using a simplified stick model and earthquake loading. Acceleration time histories were produced for the nodes at the top and bottom of the stick for the horizontal (x) and vertical (z) directions. In order to produce the peak frequency, the secondary response spectra were obtained from the acceleration time histories. Results indicated small differences between the smaller soil domain and the largest one. While more significant effects were seen due to mesh size and mass scaling. This is aligned with the expectations of using the effective seismic method in combination with PML boundaries.

SSSI LS-DYNA MODEL

Once the SSI model of the S building is established, a full model including both M building and the S building can be modelled. The SSSI model, including both buildings, using 3D sticks representation, is shown in Figure 9. The interface between the soil and the raft of each building is based on the Bielak et al. method described above.

The analysis is undertaken using best estimate (BE) degraded soil properties (as provided by the client) and a number of seismic motions, resulting in several different analyses runs.

As mentioned in Bolisetti and Whittaker (2015), the runtimes are significant for the SSI and SSSI analysis using the LS-DYNA software package, which is time-domain based. The final SSSI model used in this study, consisted of a combination of complex sticks representing the buildings, including both M building and S building. This model could take up to 20 days to run using a multi-core processors. It is worth noting that run times are also affected by the level of output data required from the analysis. In comparison to 2D SuperFLUSH analysis this is an increase of more that 480 times. No direct comparison to 3D SASSI was considered in these studies, but as per Bolisetti and Whittaker (2015), it is about 120+ times faster.

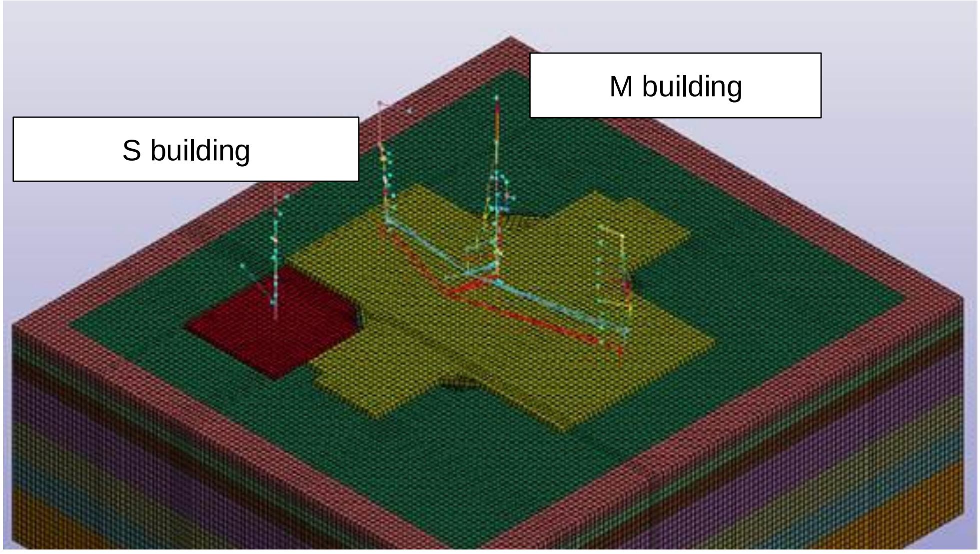


Figure 9. Two buildings SSSI model in LS-DYNA

Results

Secondary Response Spectra (SRS)

The output from both SSI and SSSI analysis is extracted as a form of acceleration time histories at different points at the base of the sticks for the S building and the M building. Subsequently these are converted into secondary response spectra and compared between the two models. A typical comparison of the results, averaged for all considered earthquake motions, is presented in Figure 10. The left hand side presents the results at the base of S (S only model vs S&M model), and the right hand side presents the results at the base of M (M only model vs S&M model) in the X, Y, and Z directions, respectively.

It needs to be noted that the mass of S building is assumed as 10% of the total mass of M building. Therefore, as expected the S building has very little SSSI effect on the M building as shown on the right hand side graphs.

Conversely, the M building does have some SSSI effect on S building as can be seen on the left hand side graphs. This is in line with clause 5.1.5 of ASCE4-16, where SSSI effects should be considered for a ‘*somewhat light structure in close proximity to a massive structure*.’ However, the SSSI effects in the horizontal directions (X and Y) indicate reduction in the accelerations, with relatively small increases in the vertical direction. The limited SSSI effects are attributed to the competent rock assumed in this studies and are in line with general best practice (for example Bolisetti and Whittaker (2015)).

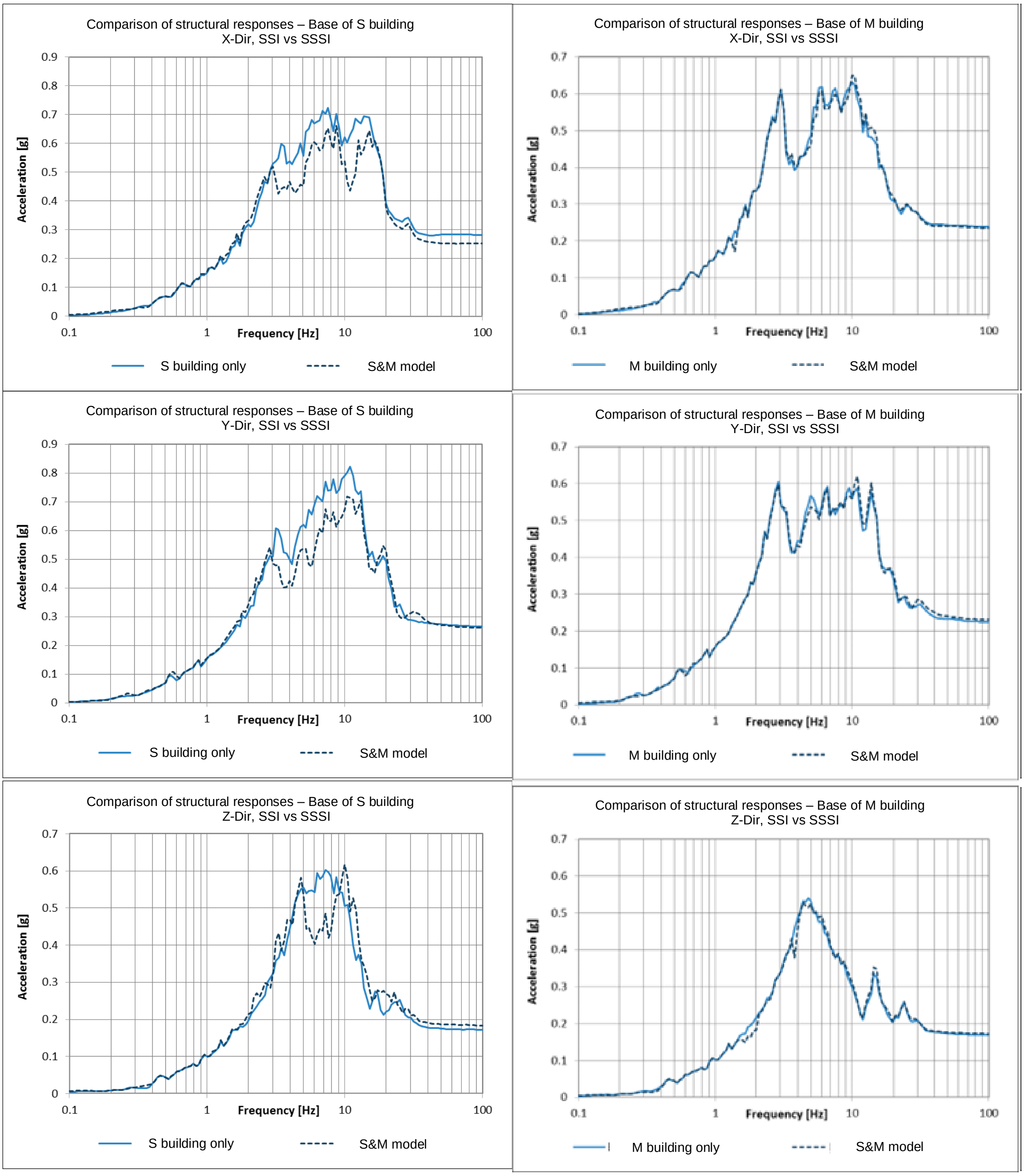


Figure 10. Comparison of SSI and SSSI response spectra at the base of M building (left) and at the base of the S building (right)

Soil Strains

A typical cross-section cut through the M building and underlying soil is shown in Figure 11 with corresponding effective Von Mises soil strains for the cross-sectional cut presented in Figure 12. Effective soil strains in plan-view (X-Y plane) are shown in Figure 13. The theoretical background to the presented soil strains can be found in the LS-DYNA Theory manual 'Hallquist, J., (2006)'.

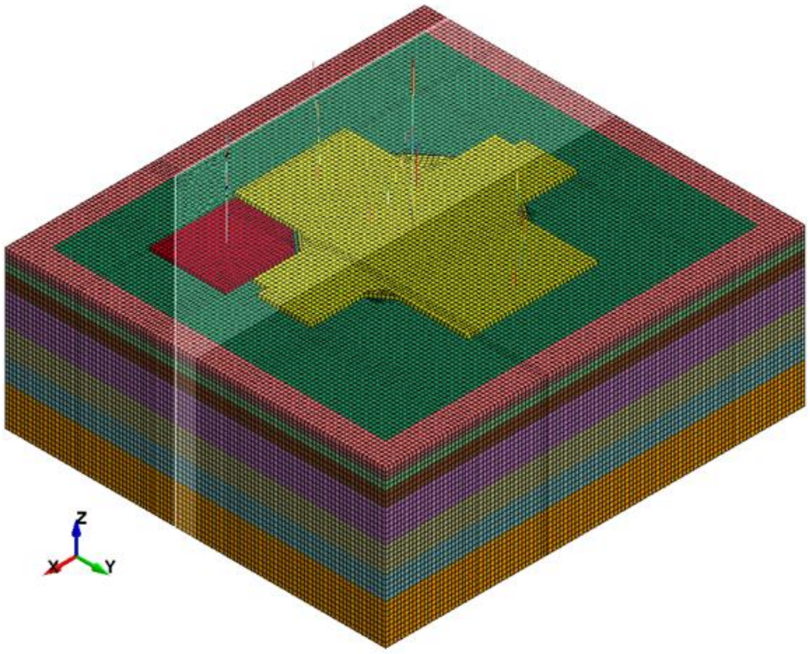


Figure 11. Location and orientation of cross-sectional cut #1

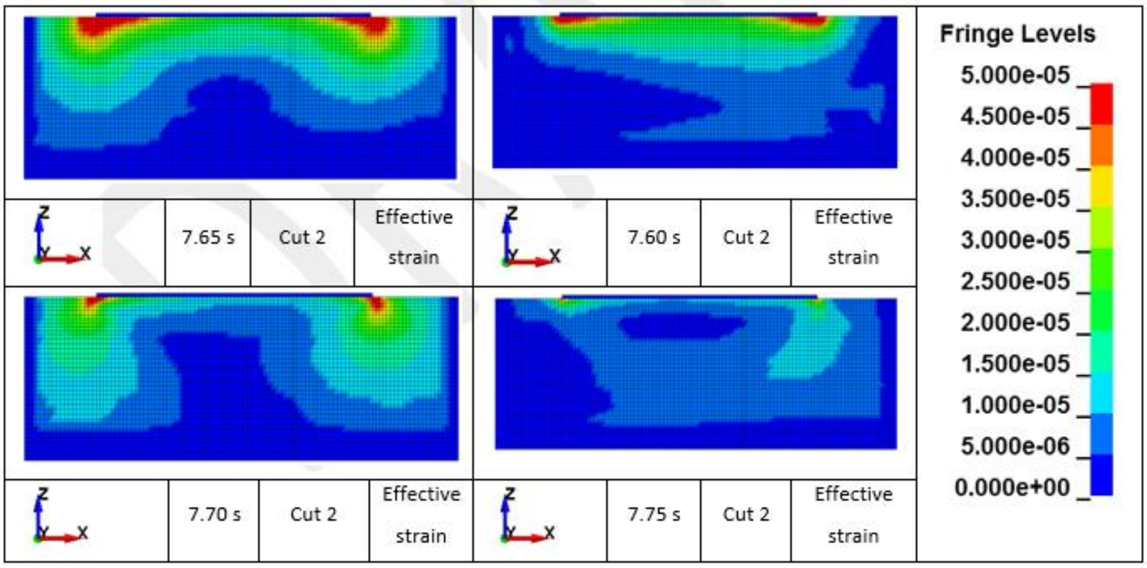


Figure 12. Effective soil strains for cross-sectional cut #1 at critical time-steps

The effective soil strain distributions are consistent with expected SSI interaction, with the maximum soil strains close to the structural edges, propagating outward and dissipating with increasing distance away from the structure. Minimal effective soil strains are visible close to the PML boundaries as expected, due to their absorbing properties.

Verification

As briefly mentioned in the introductory sections, the SSI and SSSI project encompassed several different aspects, culminating in the SSSI analysis of the S building. Prior to that a benchmarking exercise of the SSI approach was undertake, which was followed by 2D studies of the discussed studies in SuperFLUSH. During the process of undertaking the SSSI LS-DYNA study, all SSI results were cross-checked against equivalent 2D SuperFLUSH results, with a very good agreement. The SSSI results were only verified through checking and comparison to the literature, like Bolisetti and Whittaker (2015) and ASCE 4-16. However, the whole process built significant level of confidence in the results.

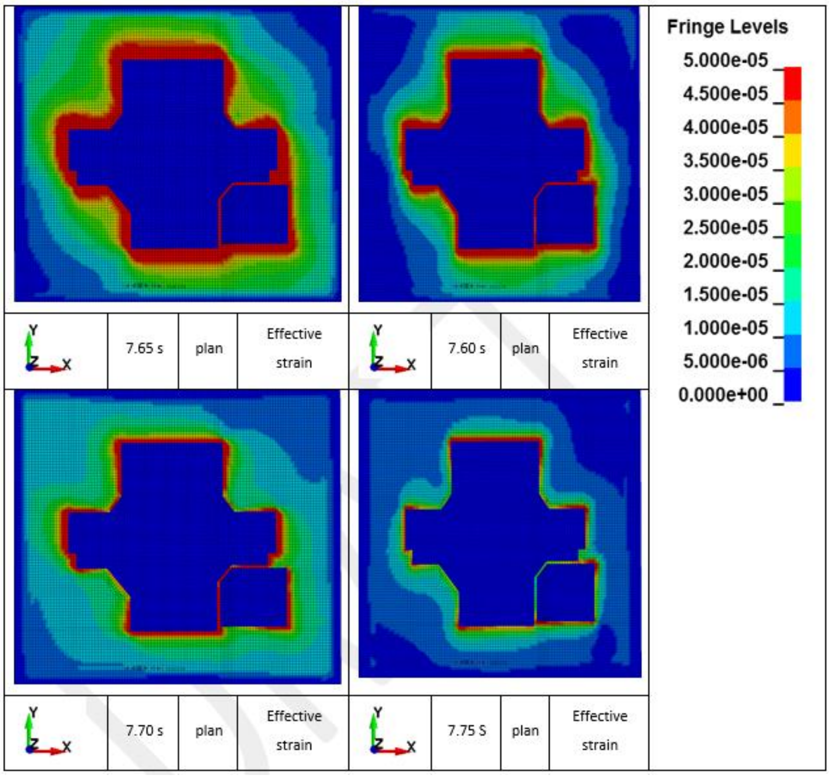


Figure 13. Effective soil strains for plan-view (X-Y plane)

Conclusion

This paper presents the innovative structure-soil-structure interaction approach for the generic new nuclear build power station, within the UK context. It focuses on a single aspect of interaction between the M building and the S building, with the effects from the bigger building onto the smaller one as the main aspect.

This paper discusses LS-DYNA software package as an SSI and SSSI tool, highlighting all the capabilities and necessary theoretical principles, but also the limitations. The important LS-DYNA cards required for a simple SSSI analysis are also listed within the appendix.

The modelling is based on a simplified approach of using stick representations of the actual buildings, capturing only the global behaviour.

Due to the comparative nature of the study, the results are based on the secondary response spectra, which can be relatively reviewed and compared. Additional discussion is presented on the development of the aggravation factors, which are used as an indication on the level of SSSI phenomenon affecting the considered building.

The conclusions of the study are that the S building seems not to be grossly affected by the presence of neighbouring M building. The results generally indicate only limited modification to the building motion in comparison to the single building SSI analysis.

General conclusions of this paper are that LS-DYNA is a very powerful tool for undertaking SSI and SSSI studies, but the analysis times are significant and need to be taken into account when choosing the best tool for the job.

Appendix – LS-DYNA commands and keywords

The primary keywords required for SSI and SSSI in LS-DYNA are summarised below and are extracted from LSTC (2014-1).

- SSI and seismic input: *INTERFACE_SSI_ID – these cards create a tied contact soil structure interface for the use in a transient analysis of a system subjected to a time history loading. *LOAD_SEISMIC_SSI – this card specifies earthquake ground motion on soil-structure interface.
- PML absorbing boundary: *MAT_PML_ELASTIC – this card assigns properties of perfectly matched layer (PML) used for transmitting boundaries
- Damping cards: *DAMPING_FREQUENCY_RANGE, provides constant damping for a specific frequency range. *DAMPING_PART_STIFFNESS_SET, is the stiffness part of Rayleigh damping. And, *DAMPING_PART_MASS_SET, is the mass part of Rayleigh damping.
- Raft: For the foundation slabs the *MAT_RIGID was used – deformations and stresses are not produced for the elements that are rigid, and therefore they are computationally very cost effective.

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