

REVIEW OF THE EQUIVALENT STATIC METHOD FOR SEISMIC ANALYSIS

Andreas NIELSEN¹ & Ian WARD²

Abstract: *In seismic analysis, the equivalent static method (ESM) is an established and relatively simple way of estimating the seismic response of structures, systems and components subject to earthquake excitation. This paper reviews various code provisions for the application of the ESM, including ASCE 4, IEEE 344, and Eurocode 8. The paper finds that there are some gaps and discrepancies in the code provisions. The paper presents a fresh introduction to the theoretical basis for the ESM, and a strict bound on the peak base shear is derived. A new set of equations for calculating an equivalent static lateral force distribution is also derived. The new equations have some advantages over the corresponding Eurocode 8 equations. Results from a numerical study based on a large set of randomised cantilevers with varying ratios of flexural stiffness to shear stiffness are presented. The study suggests that the equivalent static load factors for base shear and base moment can be set at 1.0 and 1.1, respectively. Having read this paper, the reader should have a better understanding of the ESM and feel more confident about applying the method for seismic analysis.*

Introduction

In seismic analysis, the equivalent static method (ESM) is an established and relatively simple method which can be used to estimate the response of structures, systems and components (SSCs) subject to earthquake excitation. The ESM appears under various names in codes and standards. In ASCE 4, the method is known as “equivalent static analysis”. Eurocode 8 uses the term “lateral force method”, whereas IEEE 344 (which has now merged with IEC 60980) opts for “static coefficient analysis”.

The method is traditionally considered suitable for SSCs that can be represented by a simple model. Eurocode 8, which has multi-storey buildings in mind, limits the method to buildings with “regularity in elevation” (BS EN 1998-1, §4.3.3.2.1(2)b)). ASCE 4 states that the method is applicable to SSCs with regular framing and small contributions from higher modes. The code goes on to say that the method “may also be used for simple but irregular SSCs, provided that adequate parametric studies are performed to demonstrate the adequacy of the analysis.” (ASCE 4-16, §4.5.2(b)). Unfortunately, the code does not say how the analyst should conduct the required studies without resorting to dynamic analysis.

In our experience, the method is commonly used for statically determinate SSCs that can be analysed easily by hand calculations – that is, without the use of finite element models or computer analysis. This practice is understandable because if the analyst makes the effort to build a finite element model, then he or she might as well use a dynamic method of analysis with little extra effort and obtain more accurate results.

The main attraction of the ESM compared to other methods of analysis is the fact that the method lends itself to speedy hand calculations. Therefore, any attempt to refine the method must preserve this simplicity. There is no need for an “advanced ESM” requiring computer analysis because we already have much more accurate computational methods at our disposal.

Many practitioners will routinely apply a “multi-mode factor” when using the ESM. The factor appears in ASCE 4 where, perhaps confusingly, it is called a “dynamic amplification factor” and has a value of 1.0 for systems with a single support, or 1.5 for systems with multiple supports and 1.0 for supports of such systems (ASCE 4-16, §4.5.1.4). The factor also appears in IEEE 344, where the recommended value is 1.5 (BS EN IEC/IEEE 60980-344, §11.2.3). In our experience, a factor of 1.5 can be unduly conservative when the stated limitations of simplicity and regularity

¹ Principal Engineer, Atkins, Glasgow, UK

² Technical Director, Atkins, Epsom, UK

are respected. In fact, as this paper demonstrates, there is no theoretical justification for a “multi-mode factor” when the ESM is used to calculate total base shear.

ASCE 4 also specifies that for “cantilever structures or components, the resultant force shall be assumed to act at a distance of 1.2 times the distance from the base to the center of mass.” (ASCE 4-16, §4.5.1.5). According to ASCE 4, the 1.2 factor is another multi-mode factor.

Having specified the resultant force and the point at which the result force acts, the analyst will in some cases want to convert the resultant force into a lateral force distribution in order to calculate displacements and other action effects such as internal bending moments and shear forces. This lateral force distribution should be defined such that it gives the correct values of base shear and base moment. Unfortunately, ASCE 4 and IEEE 344 do not provide much guidance in this regard (§4.5.1.6 in ASCE 4-16 apparently concerns the horizontal distribution of forces at any given floor level). Eurocode 8 defines a lateral force distribution, but the equations given in the code (BS EN 1998-1, Equations (4-10) and (4-11)) are not guaranteed to give the desired base moment – in other words, the resultant force will not act at the desired height.

The provisions in ASCE 7 are also of interest (ASCE/SEI 7-16, §12.8). A detailed review of these provisions has not been undertaken. However, it seems that the provisions are similar to Eurocode 8 provisions. It is noteworthy that the equations for the lateral force distribution include a period-dependent exponent, which is not found in Eurocode 8.

It would appear, therefore, that the ESM is an established codified method, albeit with some gaps and discrepancies in the various code provisions. However, the theoretical basis for the method is not covered by the codes. Textbooks on seismic analysis also tend to skip the method in favour of more advanced dynamic methods of analysis. For that reason, the theoretical basis remains obscure to many practitioners and undergraduates. The aim of the present paper is to rectify that and review the theoretical basis for the ESM.

This paper also presents a new numerical study of a cantilever. A “cantilever” in this context should be understood in a wide sense; it can be any SSC ranging from a simple pipe support to a multi-storey building, provided the SSC in question can be simplified as series of one-dimensional line elements with discrete masses.

Theoretical basis

Fundamental considerations

The fundamental basis for the ESM is a simple theorem which states that the centre of gravity (CoG) of a system of particles moves as a single particle with a mass equal to the total mass of the system and subject to the same external forces as the system (Alonso & Finn, 1980). In mathematical terms:

$$M\mathbf{a}_c(t) = \mathbf{F}_{\text{ext}}(t) \quad (1)$$

where M is the total mass, $\mathbf{a}_c(t)$ is the acceleration of the CoG and $\mathbf{F}_{\text{ext}}(t)$ is the sum of external forces acting on the system. In the general case, \mathbf{a}_c and \mathbf{F}_{ext} are both vectorial quantities (which in this paper are denoted by bold, upright typeface). Static forces including gravity can be ignored without loss of generality. Thus, if we know the acceleration, we can calculate the external force.

For a linear-elastic single degree-of-freedom (DOF) system, Equation (1) becomes the well-known ordinary differential equation governing such systems. The CoG coincides with the position of the one mass, and so, when the system is subject to base acceleration, the peak acceleration can be obtained directly from a response spectrum. This gives us the desired force.

Equation (1) is valid for a multi DOF system, but it does not give us sufficient information about the response of the system to completely determine the action effects in the system. Nevertheless, the ESM is based on the observation that the static deformed shape under external gravity-type loading (horizontal or vertical) often resembles the shape of the first natural mode of a multi DOF system. As a result, the equivalent static action effects are an acceptable approximation to the peak dynamic action effects when the response is governed by the first mode. However, the validity of the approximation can deteriorate when the higher modes have a marked influence on the response of the system, and locally the true dynamic action effects can

exceed the equivalent static action effects. This is the reason why codes specify a so-called multi-mode factor, which is intended to account for the effects of higher modes.

For a structure subject to base excitation, the external force \mathbf{F}_{ext} is the base shear \mathbf{V}_b . The latter term is slightly misleading because \mathbf{F}_{ext} also comprises vertical force. However, “base shear” is a term that is familiar to earthquake engineers. In the following, we derive a rigorous bound on the base shear.

Base shear

The response of the structure is assumed to be linear-elastic and classically damped. This means that the structure will possess a set of real natural modes. Let n denote the mode number, and N the total number of modes. Then, adopting the annotation of Chopra (2001), the base shear at time t due to the response of mode n is:

$$\mathbf{V}_{bn}(t) = V_{bn}^{\text{st}} \mathbf{A}_n(t) \tag{2}$$

where $\mathbf{A}_n(t) = [A_{nx}(t), A_{ny}(t), A_{nz}(t)]^T$ is the vector of pseudo-acceleration responses of the n th mode, and V_{bn}^{st} is a matrix of modal static base shears (see Chopra, 2001, for an explanation of these terms):

$$V_{bn}^{\text{st}} = \begin{bmatrix} V_{bnxx}^{\text{st}} & V_{bnxy}^{\text{st}} & V_{bnxz}^{\text{st}} \\ V_{bnyx}^{\text{st}} & V_{bnyy}^{\text{st}} & V_{bnyz}^{\text{st}} \\ V_{bnzx}^{\text{st}} & V_{bnzy}^{\text{st}} & V_{bnzz}^{\text{st}} \end{bmatrix} \tag{3}$$

Within this matrix, the last two letters in each subscript denote force in the p direction due to excitation in the q direction (with $p, q = x, y, z$); for example, V_{bnyz}^{st} is modal static base shear in the y direction due to excitation in the z direction.

Without proof, we postulate that the off-diagonal elements in V_{bn}^{st} are equal to zero for structures with symmetry about two vertical planes, and we now restrict our attention to structures with the described symmetry. Incidentally, the stated restriction suggests that the ESM should be limited to symmetric structures.

Having excluded directional coupling, we can focus on a single direction (say, x). The modal static base shear V_{bnxx}^{st} is equal to the effective modal mass M_{nx}^* – again, the reader is referred to Chopra (2001) for details. Hence, summing all modal values and using Equation (1), we obtain for direction x :

$$V_{bx}(t) = \sum_{n=1}^N M_{nx}^* A_{nx}(t) = M a_{cx}(t) \tag{4}$$

where a_{cx} is the x -component of \mathbf{a}_c . We define the envelope process for direction x as,

$$A_x^{\text{env}}(t) = \max \left\{ |A_{1x}(t)|, |A_{2x}(t)|, \dots, |A_{Nx}(t)| \right\} \tag{5}$$

By definition, the peak spectral acceleration S_{Ax}^{peak} constitutes an upper bound on the envelope process:

$$S_{Ax}^{\text{peak}} \geq A_x^{\text{env}}(t) \quad \forall t \tag{6}$$

The pseudo-acceleration response of the n th mode may be written as a fraction of the envelope process:

$$A_{nx}(t) = \alpha_{nx}(t) A_x^{\text{env}}(t) \quad \text{with} \quad -1 \leq \alpha_{nx}(t) \leq 1 \tag{7}$$

Inserting (7) into (4), taking absolute values, and re-arranging, we find that:

$$|a_{cx}(t)| = \frac{1}{M} \left| \sum_{n=1}^N M_{nx}^* \alpha_{nx}(t) \right| A_x^{\text{env}}(t) \quad (8)$$

We know that that the sum of effective modal masses is equal to the total mass of the structure, and that $M_{nx}^* \geq 0$. For that reason, we must have:

$$\left| \sum_{n=1}^N M_{nx}^* \alpha_{nx}(t) \right| \leq M \quad (9)$$

Using (6), (8) and (9), the following bounds are derived:

$$|a_{cx}(t)| \leq A_x^{\text{env}}(t) \leq S_{Ax}^{\text{peak}} \quad (10)$$

Since the acceleration of the CoG is always less than or equal to the peak spectral acceleration, we can use the following equation to calculate the peak base shear:

$$V_{bx} = \lambda_V M S_{Ax}^{\text{peak}} \quad (11)$$

where λ_V is the equivalent static load factor for base shear. Based on the bounds of Equation (10), we conclude that

$$\lambda_V \leq 1 \quad (12)$$

We now turn our attention to the base moment.

Base moment

The external moment \mathbf{M}_{ext} acting on a system of particles is given by another theorem of mechanics which states that the external moment can be written as a sum of two terms (Alonso & Finn, 1980):

$$\mathbf{M}_{\text{ext}} = M \mathbf{a}_c \times \mathbf{r} + \frac{d\mathbf{L}_c}{dt} \quad (13)$$

In this equation, we omitted the time dependence of all vectors, but they are implied. The vector \mathbf{r} is the distance from an inertial frame of reference to the CoG; and \mathbf{L}_c is the angular momentum of the particles about a coordinate system that is fixed to the CoG.

Again, for a structure subject to base excitation, the external moment \mathbf{M}_{ext} is the base moment \mathbf{M}_b . Equation (13) tells us that the base moment is not necessarily bounded by an equivalent static force times a lever arm from the base to the CoG. Depending on the time variation of \mathbf{L}_c , the base moment may be less or greater than the moment resulting from the cross-product of $M \mathbf{a}_c$ and \mathbf{r} .

In the following we restrict the investigation to vertical cantilevers and consider again only one direction of excitation. We choose the y -axis as the vertical axis, and we define y_c as the distance from the base to the CoG. We are interested in estimating the peak base moment about the z -axis, M_{bz} . In the ESM, it is convenient to compute the base moment according to the following equation:

$$M_{bz} = \lambda_M M y_c S_{Ax}^{\text{peak}} \quad (14)$$

where λ_M is the equivalent static load factor for base moment. It is easy to evaluate this factor accurately based on dynamic analysis. Equation (13) suggests that the factor can be less or greater than unity because, as we saw above, the base moment may be less or greater than the product $M y_c S_{Ax}^{\text{peak}}$. Equation (10) implies that in many cases the factor will be less than unity.

However, we resort to the ESM because we do not want to carry out a dynamic analysis. Therefore, a conservative estimate of λ_M is required. ASCE 4-16 suggests that the maximum value is $\lambda_M = 1.2$ based on the results published by Niehoff and Gürbüz (2007).

Lateral force distribution

The lateral force resultant is equal to the base shear, which in the ESM is calculated according to Equation (11). From Equation (14), the lateral force resultant should be applied at a height $y = \lambda_M y_c$ above the base. In some situations, a more detailed lateral force distribution is required

(e.g. to calculate storey shears). Eurocode 8 (BS EN 1998-1) suggests that the lateral force distribution should be calculated as follows:

$$F_{xj} = V_{bx} \frac{m_j s_j}{\sum_j m_j s_j} \tag{15}$$

where F_{xj} is the lateral force applied at level j , and s_j is the displacement of mass m_j in the (assumed) fundamental mode shape φ . This formula gives the correct base shear. However, the formula does not, in general, give the correct base moment. The reason for this can be discerned by closer examination of the force distribution. At the base, the modal displacement φ is always zero, and as a result Equation (15) predicts zero force at the base and small forces close to the base. However, we know that masses at the base or close to the base are accelerated in an earthquake and generate force. Therefore, the lateral force distribution resulting from the first mode shape does not intentionally represent the actual inertial forces at a point in time. Instead, the equivalent static lateral force distribution should be defined such that the resulting action effects are consistent with enveloped results from more accurate, dynamic analyses. We propose that a better lateral force distribution can be found if we abandon the idea that the forces should be proportional to mode displacements. The simplest distribution is linear:

$$s(y) = ay + b \tag{16}$$

where the two parameters a and b are calculated as:

$$a = \frac{(\lambda_M - \lambda_V) My_c}{\sum_j m_j y_j^2 - My_c^2} \tag{17}$$

$$b = \lambda_V - ay_c$$

where y_j is the height of mass m_j above base.

It can be shown that Equations (15), (16) and (17) give the correct values of base shear (Equation (11)) and base moment (Equation (14)).

The question now arises: what values of λ_V and λ_M are safe to use for design purposes? The rest of this paper attempts to answer this question.

Numerical study

We consider a vertical cantilever with J discrete masses and J massless beam elements (Figure 1). The height of each beam element is h , and the total height of the cantilever is $H = Jh$. Beam element j has flexural rigidity EI_j , where E is Young’s modulus and I_j is the second moment of area, and shear rigidity GA_j , where G is the shear modulus and A_j is the shear area.

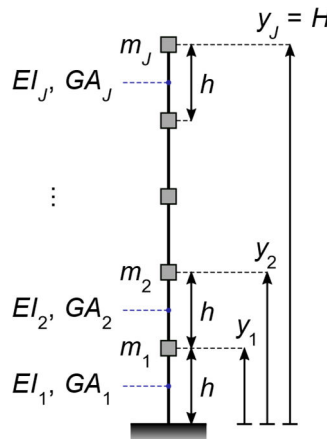


Figure 1. Definition sketch

The stiffness matrix for each beam element is given by (Krenk, 2000):

$$k_j = \frac{EI_j}{(1 + \phi_j)h^3} \begin{bmatrix} 12 & -6h & -12 & -6h \\ -6h & (4 + \phi_j)h^2 & 6h & (2 - \phi_j)h^2 \\ -12 & 6h & 12 & 6h \\ -6h & (2 - \phi_j)h^2 & 6h & (4 + \phi_j)h^2 \end{bmatrix} \quad (18)$$

where ϕ_j is the ratio of flexural stiffness to shear stiffness:

$$\phi_j = \frac{12EI_j}{GA_jh^2} \quad (19)$$

We generate 999 sets of J random values of EI and m drawn from a uniform distribution (i.e. a probability distribution) and assign the random values to 999 cantilevers. The probability distribution for EI is defined by the bounds $[10^4; 10^6]$ (units are not important), and the probability distribution for m is defined by the bounds $[10^0; 10^2]$. To the list of 999 random sets, we prepend one more set (set number 1), which represents a cantilever with a uniform spatial distribution of EI and m . We then calculate the GA values for each cantilever as:

$$GA_j = \frac{12EI_j}{\Phi H^2} \quad (20)$$

where Φ is a parameter that represents the “total” ratio of flexural stiffness to shear stiffness. For $\Phi \ll 1$ the response of the beam is dominated by flexural deformation, whereas for $\Phi \gg 1$ the response is dominated by shear deformation. From Equation (20) it follows that each cantilever is characterised by a single value of Φ (and hence ϕ).

We consider first the case for $J = 5$. A sample of EI and m values is given in Table 1.

| Set → | 1 | | 2 | | 3 | | ... | 1000 | |
|-------|-----------------------------|----------------------------|-----------------------------|----------------------------|-----------------------------|----------------------------|-----|-----------------------------|----------------------------|
| j | EI_j ($\times 10^5$) | m_j ($\times 10^1$) | EI_j ($\times 10^5$) | m_j ($\times 10^1$) | EI_j ($\times 10^5$) | m_j ($\times 10^1$) | | EI_j ($\times 10^5$) | m_j ($\times 10^1$) |
| 1 | 5.0 | 5.0 | 8.9 | 1.6 | 8.1 | 2.5 | | 7.8 | 6.4 |
| 2 | 5.0 | 5.0 | 7.3 | 7.8 | 4.0 | 2.0 | 4.4 | 0.2 | |
| 3 | 5.0 | 5.0 | 8.2 | 3.9 | 7.0 | 7.6 | 3.4 | 8.3 | |
| 4 | 5.0 | 5.0 | 7.1 | 9.8 | 6.6 | 9.4 | 5.9 | 5.3 | |
| 5 | 5.0 | 5.0 | 8.8 | 2.3 | 5.9 | 2.5 | 9.9 | 6.2 | |

Table 1. Sample of EI and m values

For the sake of simplicity, we assume that the input response spectrum is a “flat” spectrum with a constant value of spectral acceleration.

We generate 100 values of Φ equally spaced on a logarithmic scale in the interval $[10^{-4}; 10^4]$. Then, for each value of Φ and for each cantilever, we conduct a response spectrum analysis. We compute the $N = J$ natural modes and combine the modes using the CQC method (Wilson *et al*, 1981). We assume that the response spectrum method yields “exact” results. We compute the exact peak base shear and base moment. We also conduct an equivalent static analysis and calculate the values of λ_V and λ_M required to give exact results for base shear and base moment, respectively. These 100×1000 values are shown as dots in Figure 2 and Figure 3. Also shown in Figure 2 and Figure 3 are the exact λ_V and λ_M values for set 1 (orange line). Finally, we have plotted two blue lines that bound most exact values. These two lines are defined by:

$$\begin{aligned} \lambda_V &= 1.0 \\ \lambda_M &= 1.1 \end{aligned} \quad (21)$$

The equivalent static load factors defined by Equation (21) are by inspection sufficiently conservative to be used for design purposes. This in turn indicates that the values recommended by ASCE 4 ($\lambda_V = 1.0$, $\lambda_M = 1.2$) are safe upper bounds, which may be overly conservative in some situations.

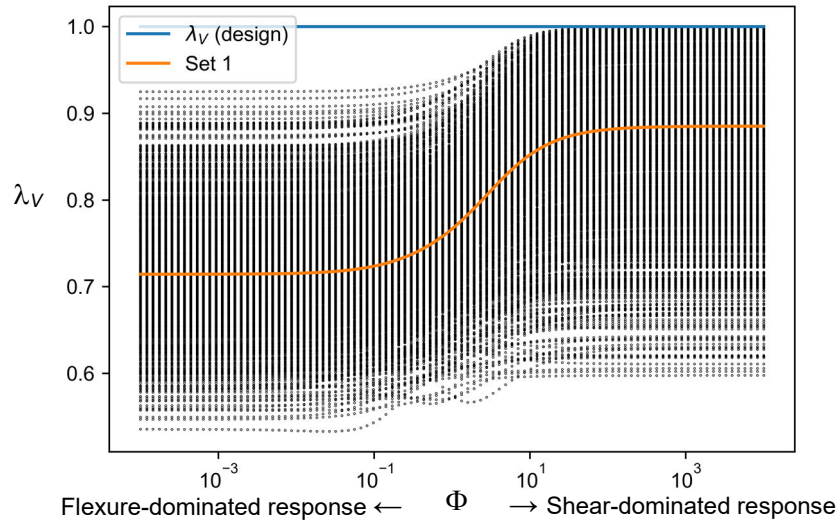


Figure 2. Equivalent static load factor for base shear ($J = 5$)

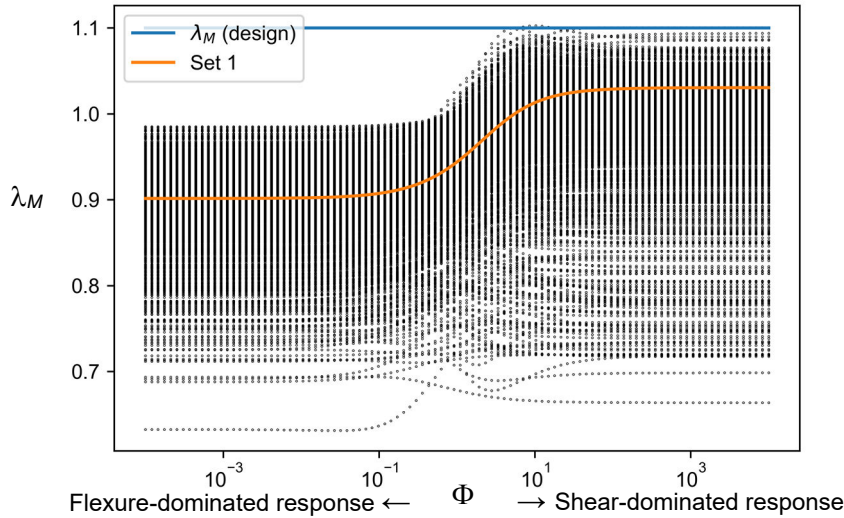


Figure 3. Equivalent static load factor for base moment ($J = 5$)

The above two figures reveal a marked correlation between the Φ value and the equivalent static load factors, with lower λ -factors for flexure-dominated cantilevers. However, while there is scope for specifying lower λ -factors for $\Phi < 1$, we consider that the added complexity is not warranted.

We also consider the tip displacement (i.e. the displacement of mass m_j). We first compute the exact displacement u_{CQC} for each value of Φ and for each cantilever. Subsequently we compute the ESM displacement u_{ESM} obtained by using the design values of λ_V and λ_M (Equation (21)) and the lateral force distribution defined by Equations (15), (16) and (17). The ratio of the two results is

$$R_u = \frac{u_{CQC}}{u_{ESM}} \tag{22}$$

For $R_u < 1$, the ESM is conservative, whereas for $R_u > 1$, the ESM is unconservative. The ratio R_u is plotted in Figure 4, which shows that the proposed ESM is conservative in all cases except for 12 cantilevers with $\Phi > 1$. The 12 cantilevers represent only 1.2% of the total population considered in this study. These few cantilevers are considered outliers that are unlikely to be encountered in practice.

We also compute the R_u ratios using the Eurocode 8 method (BS EN 1998-1, §4.3.3.2) with $\lambda = 0.85$. Admittedly not all cantilevers considered in this study satisfy the Eurocode 8 conditions for regularity in elevation (BS EN 1998-1, §4.2.3.3). Nevertheless, it is instructive to compare the

performance of the Eurocode 8 method with the method proposed in this paper. Figure 5 shows the R_u ratios obtained from the Eurocode 8 method. The Eurocode method performs very well for a shear-dominated cantilever ($\Phi > 1$) with perfect regularity in elevation (set 1), which is the type of structure the Eurocode provisions are aimed at. However, by comparison with Figure 4, we conclude that the spread of R_u ratios for any given value of Φ is wider, and that many more ratios exceed unity, indicating unconservative results. Nevertheless, had we used $\lambda = 1.0$ in the Eurocode method, all R_u ratios would have been less than unity.

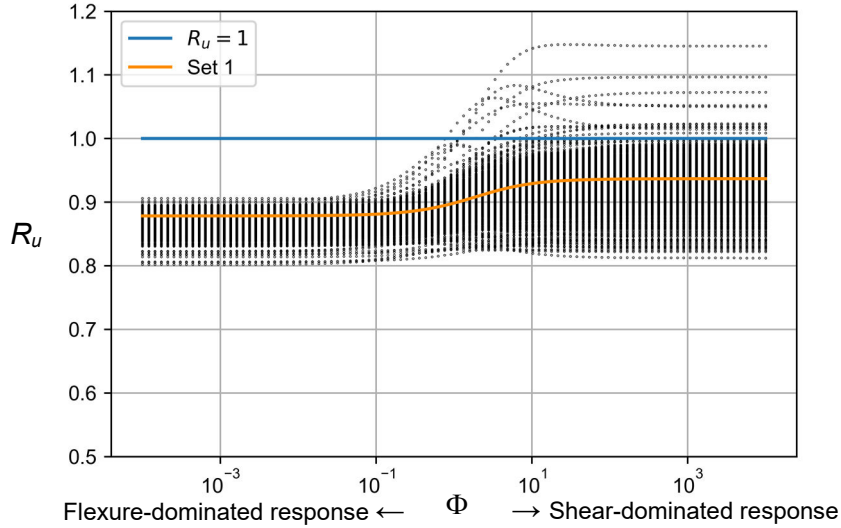


Figure 4. Displacement ratio ($J = 5$)

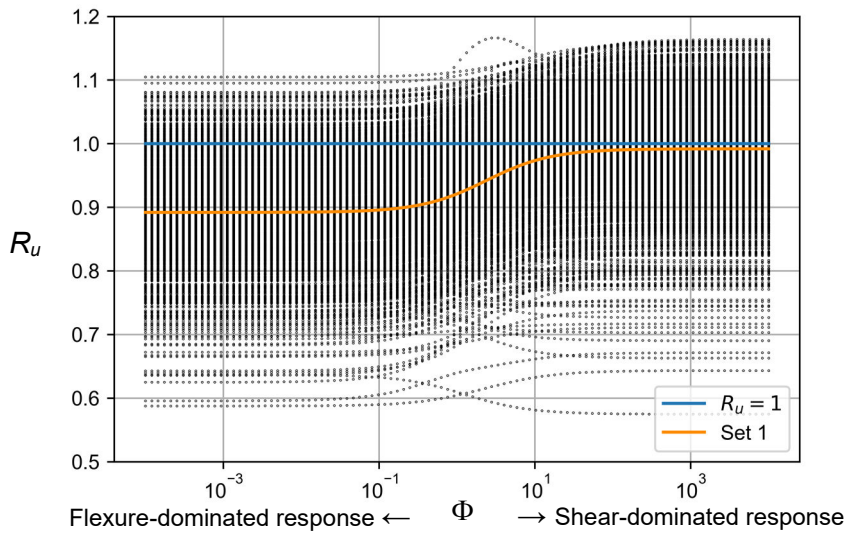


Figure 5. Displacement ratio using Eurocode 8 method ($J = 5$)

We repeat the above analyses for $J = 2, 10$ and 15 , and based on those analyses we conclude:

- The recommended design values for λ_V and λ_M (Equation (21)) remain sufficiently conservative.
- The recommended lateral force distribution (Equations (15)-(17)) continue to perform better in terms of predicting tip displacement than the Eurocode method. For $\Phi < 1$, the recommended ESM always returns a conservative estimate of tip displacement. For $\Phi > 1$, only a few cases are unconservative with $R_u > 1$.
- For $J = 2$, the maximum λ_V and λ_M factors approach unity for all values of Φ , as would be expected since at $J = 1$ (the single DOF system), both λ_V and λ_M must equal unity.
- For $J = 10$ and $J = 15$, the maximum λ_M factors remain reasonably constant for all values of Φ , whereas the maximum λ_V factors remain constant for $\Phi > 1$ and decrease to about 0.85 at $J = 10$, and to about 0.80 at $J = 15$, for $\Phi < 1$.

Summary and Conclusions

This paper examined the equivalent static method (ESM) as a tool for seismic analysis of simple structures. The paper reviewed various code provisions for the application of the ESM, including ASCE 4, Eurocode 8 and IEEE 344. The paper found that there are some discrepancies between the various methods. In the case of Eurocode 8, the ESM is clearly targeted at regular multi-storey buildings, and the Eurocode 8 provisions seem suitable for that purpose. However, the Eurocode 8 method cannot be used beyond its stated limitations.

The paper reviewed the theoretical basis for the ESM. A strict bound on the peak base shear was derived. This bound is valid for all linear-elastic systems with classical damping, provided the system in question displays symmetry about two perpendicular planes. The paper also reviewed the equations governing base moment and concluded that the effective lever arm for calculating base moment may be greater than or less than the distance from the base to the centre of gravity. However, a strict bound on the base moment was not derived in this paper.

The paper presented a selection of results from a numerical study based on a large set of randomised cantilevers with varying ratios of flexural stiffness to shear stiffness. The study was similar to the one presented by Niehoff & Gürbüz (2007), but with a much larger data set. The study suggested that the equivalent static load factors for base shear and base moment can be safely set at 1.0 and 1.1, respectively.

The paper presented a new set of equations for calculating an equivalent static lateral force distribution. The new equations have two advantages over the corresponding Eurocode 8 equations:

- They always give the desired base moment (unlike the Eurocode 8 equations).
- They yield improved predictions of peak displacement.

It should be noted that the equations for lateral force distribution are not unique. Other functional forms could have been chosen, and further research may yet discover an improved function that yields more precise and yet conservative action effects.

In conclusion, having read this paper, the reader should have a better understanding of the ESM and feel more confident about applying the method for seismic analysis of simple structures, systems and components.

References

- Alonso, M & Finn, EJ (1980), *Fundamental University Physics, Volume I: Mechanics & Thermodynamics*, 2nd Ed, Addison-Wesley.
- ASCE/SEI 4-16, *Seismic Analysis of Safety-Related Nuclear Structures and Commentary*, American Society of Civil Engineers 2017.
- ACSE/SEI 7-16, *Minimum Design Loads and Associated Criteria for Buildings and Other Structures*, American Society of Civil Engineers 2017.
- BS EN 1998-1:2004+A1:2013, *Eurocode 8: Design of structures for earthquake resistance – Part 1: General rules, seismic actions and rules for buildings*, BSI 2013.
- BS EN IEC/IEEE 60980-344:2021, *Nuclear facilities – Equipment important to safety – Seismic qualification*, BSI 2021.
- Chopra, AK (2001), *Dynamics of Structures, Theory and Applications to Earthquake Engineering*, 2nd Ed, Prentice Hall.
- Krenk, Steen (2000), *Mechanics and Analysis of Beams, Columns and Cables*, Polyteknisk Press, 1st Ed.
- Niehoff, D & Gürbüz, O (2007), “Multi-mode factor for cantilevered structures with variable mass and stiffness”, *Transactions SMiRT 19*, Toronto.
- Wilson, EL; Der Kiureghian, A & Bayo, EP (1981), “A replacement for the SRSS method in seismic analysis”, *Earthquake Engineering & Structural Dynamics*, Vol. 9, 187-194