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PROBABILISTIC ANALYSIS OF THE RESPONSE OF PLATES SUBJECTED TO NEAR-FIELD BLAST LOADING

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Abstract: *Accurate prediction of the response of structures subjected to close proximity blast loads is a pressing engineering concern; the landscape of global terror has shifted away from large and indiscriminate bombings towards much smaller and more targeted attacks (e.g. against critical infrastructure and/or transport). In such close-proximity blast events (in the so-called 'nearfield'), interaction between the expanding detonation products and air shock gives rise to complex hydrodynamic features which introduce localised variations in the pressure field. The resultant loading (typically defined in terms of specific impulse since loading durations act on timescales considerably shorter than structural response) is therefore highly uncertain, and even nominally identical experiments produce loading distributions with a high degree of local variability. Current predictive approaches either grossly simplify or neglect entirely the inherent 'fuzziness' of nearfield blast loading, to the extent where it is currently unknown what effect this has on structural response, how sensitive plate structures are to uncertainties in loading distribution, and how this varies with plate properties and loading condition (e.g. charge mass and stand-off distance). This paper presents a numerical study aimed at answering these questions, where specific impulse distributions are probabilistically simulated with varying degrees of localised variations and mapped onto a range of different plates. This work aims to shed light on the fundamentally stochastic nature of close-proximity blast, with a view to implementing the findings in fast running engineering models for prediction of plate response under near-field blast loading.*

Introduction

If blast protection engineers are to protect structures against targeted explosive attacks then the tools need to be available for predicting structural response to close proximity loading. It is particularly important in the near-field where the magnitude of pressure is greatest and time duration of loading is small (Tyas, 2019). It is in this region that attackers can cause significant damage to key structural members or protective apparatus.

During near-field loading the expanding detonation product cloud (DPC) is still in contact with the blast wave (Tyas, 2019). In an idealised scenario a target in close proximity to an explosive charge will experience a centrally localised specific impulse distribution (Pannell *et al.*, 2021). However, interactions between the DPC and shock wave result in a hydrodynamically complex environment.

During the early/extreme near-field, loading is consistent with little variation (S E Rigby *et al.*, 2020; Pannell *et al.*, 2021). However, as scaled distance increases into the late near-field, hydrodynamic structures such as Rayleigh-Taylor (RT) and Richtmyer-Meshkov (RM) instabilities develop and grow (Fuller *et al.*, 2016; Tyas *et al.*, 2016; S. E. Rigby *et al.*, 2020). These instabilities are formed at the fluid interface between the DPC and shock wave, where a light fluid (DPC) accelerates into a heavy fluid (compressed air within the shock wave) (Zhou *et al.*, 2021) and can form protrusions within the fireball. These hydrodynamic structures give rise to spatially complex and variable blast loading that is stochastic in nature (S E Rigby *et al.*, 2020). When impinging on

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a plate this can result in off-center or asymmetric loading of the plate, as well as a locally increased specific impulse.

Testing was performed by Rigby *et al.* (2019) on cylindrical and spherical charges at two scaled distances (to account for directional loading of cylindrical charges). The study noted a marked increase in variability and loading offset in the case of the larger stand-off cylindrical charges, which was attributed to the formation of RT and RM instabilities. Balakrishnan *et al.* (2010) found that the formation of these instabilities is exacerbated by charge surface imperfections, meaning two nominally identical tests can result in vastly different loadings.

These complicated interactions between the DPC and shock wave make near-field blast loading inherently variable and stochastic. If protective engineers are to accurately predict blast loading in the near-field, then they must fully account for this intrinsic variability. In order to do this, fastrunning predictive models should move away from a deterministic approach and instead employ probabilistic methods to assess loading and structural response. Only in this way can the full range of loading and deflection outcomes be captured.

This paper presents a series of Monte Carlo analyses of structural response using numerical modelling to determine structural plate response to variable loading profiles impinging upon it. In this way several factors of variability are assessed by their effect on plate response variability.

Method of Analysis

In order to perform a numerical study of a near-field blast loaded plate, models of both loading distribution and the resultant structural response were employed. The models used to determine these parameters are described below.

Specific Impulse Model

To evaluate the specific impulse distribution applied across the plate or target structure, a loading model established by Pannell *et al.* (2021) was used. This model is shown in equation 1.

$$i(Z,\theta,W) = 0.557Z^{-1.663}e^{-2007\underline{\quad}\theta}W^{1/3} \tag{1}$$

For $0.11 \leq Z \leq 0.55$ m/kg$^{1/3}$, where i – specific impulse (MPa.ms); Z – scaled distance (m/kg$^{1/3}$) ; θ – angle of incidence (°); W – charge mass (kg).

The model developed by Pannell *et al.* (2021) describes an idealised specific impulse profile where the localisation centre coincides with the centroid of the plate. To model asymmetric loading within this study, the localisation centre was offset by altering the angle of incidence coordinate system. All specific impulse distributions in this study were evaluated using a charge mass of 80g.

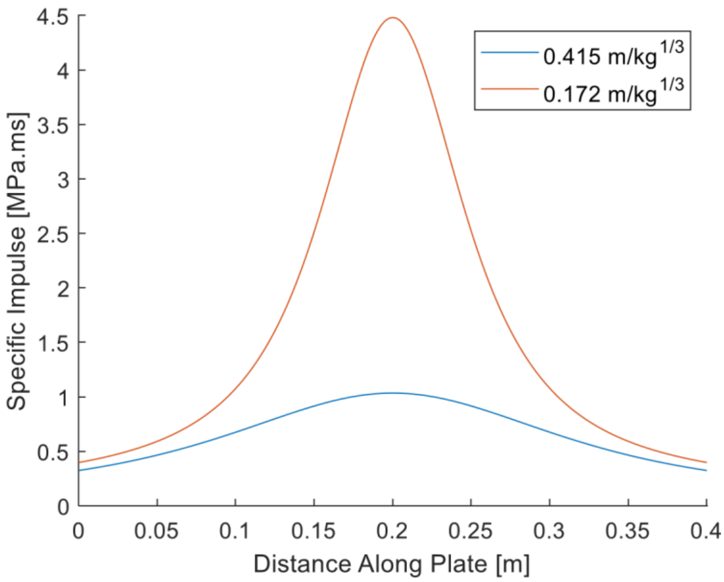

Figure 1: Specific impulse distributions derived using model presented by Pannell et al. (2021) at both scaled distances used within this study.

Numerical Model

A lagrangian finite element model was created using LS-DYNA (Hallquist, 2006), to model a 400x400x3 mm square DOMEX 355 MC steel plate with a mesh density of 2 mm. All four sides of the plate were restrained against all translation and rotation. The plate was modelled using a simplified Johnson-Cook material model of the same parameters and properties as those used in studies by Rigby *et al.* (2019) and Rigby *et al.* (2021).

The specific impulse distribution impinging on the plate was converted to initial velocities and applied at each node using the *INITIAL_VELOCITY_NODE keyword. The results were then used to determine the maximum displacement of the plate, z_{max}.

Monte Carlo Analyses

Due to the inherent variability and stochastic nature of near-field blast loading, a probabilistic approach has been used to define the maximum displacement of a plate in close-proximity to an explosive charge. A series of Monte Carlo analyses have been undertaken, where variable input parameters are sampled from a pre-determined probability distribution which describes both the magnitude and variability of that parameter. Each sample input is run through the specific impulse and LS-DYNA plate models described above. By running multiple sample models and recording the z_{max} results, the output probability distribution can be determined for the given input distribution.

In this family of Monte Carlo analyses, three variable input parameters are used:

- Scaled Distance
- Localisation Centre Offset in the x direction
- Localisation Centre Offset in the y direction

Analyses are performed at two scaled distances (0.172 m/kg$^{1/3}$ and 0.415 m/kg$^{1/3}$) and each variable input parameter is sampled from a normal distribution. Two different normal distributions are used for each input parameter to represent “Small” and “Large” variation of loading. The distribution parameters are shown in Table 1.

Input Parameter	**Variation**	**Mean Value, μ**	**Standard Deviation , σ**	**Normalised Stan Deviation σ/μ or σ/Plate Width**	**Type**
Scaled Distance, Z	Large	0.172 or 0.415 m/kg$^{1/3}$	0.166 m/kg$^{1/3}$	0.4	Normal
	Small	0.172 or 0.415 m/kg$^{1/3}$	0.021 m/kg$^{1/3}$	0.05	Normal
	Fixed	0.172 or 0.415 m/kg$^{1/3}$	-	-	-
x Offset	Large	0 mm	80 mm	0.2	Normal
	Small	0 mm	20 mm	0.05	Normal
y Offset	Large	0 mm	80 mm	0.2	Normal
	Small	0 mm	20 mm	0.05	Normal

Table 1: Parameters for input probability distributions used in Monte Carlo analyses.

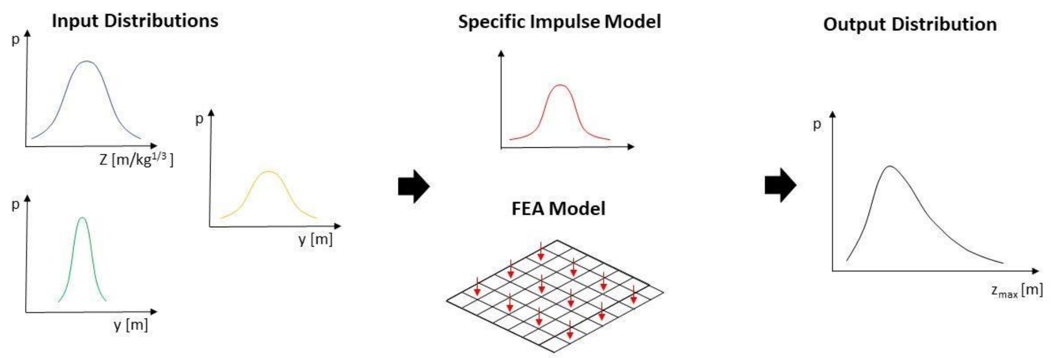

Figure 2: Illustration of Monte Carlo analyses undertaken in this study.

Two separate families of Monte Carlo Analyses were performed:

- Set A – values of offset in the x and y direction are sampled from the Small or Large probability distributions and are therefore variable for each sample model. Scaled distance is kept constant for each sample model (i.e. either 0.415 or 0.172 m/kg$^{1/3}$).
- Set B - Offset in x and y direction are kept variable as in Set A, whilst scaled distance is sampled from either the Small or Large distributions and is therefore variable for each sample model.

In both Set A and B, Monte Carlo analyses are performed for "Large" and "Small" variation of the input parameters at both scaled distances. This results in a total of 8 Monte Carlo analyses, as summarised in Table 2.

Set	Analysis	Sample Models	Z [m/kg$^{1/3}$]		X Offset [mm]		Y Offset [mm]	
			μ_z	σ_z	μ_x	σ_x	μ_y	σ_y
A	i	30	0.415	-	0	80	0	80
	ii	30	0.415	-	0	20	0	20
	iii	30	0.172	-	0	80	0	80
	iv	30	0.172	-	0	20	0	20
B	v	30	0.415	0.166	0	80	0	80
	vi	30	0.415	0.021	0	20	0	20
	vii	30	0.172	0.166	0	80	0	80
	viii	30	0.172	0.021	0	20	0	20

Table 2: Summary of Monte Carlo Analyses performed.

Results and Discussion

Analysis Set A

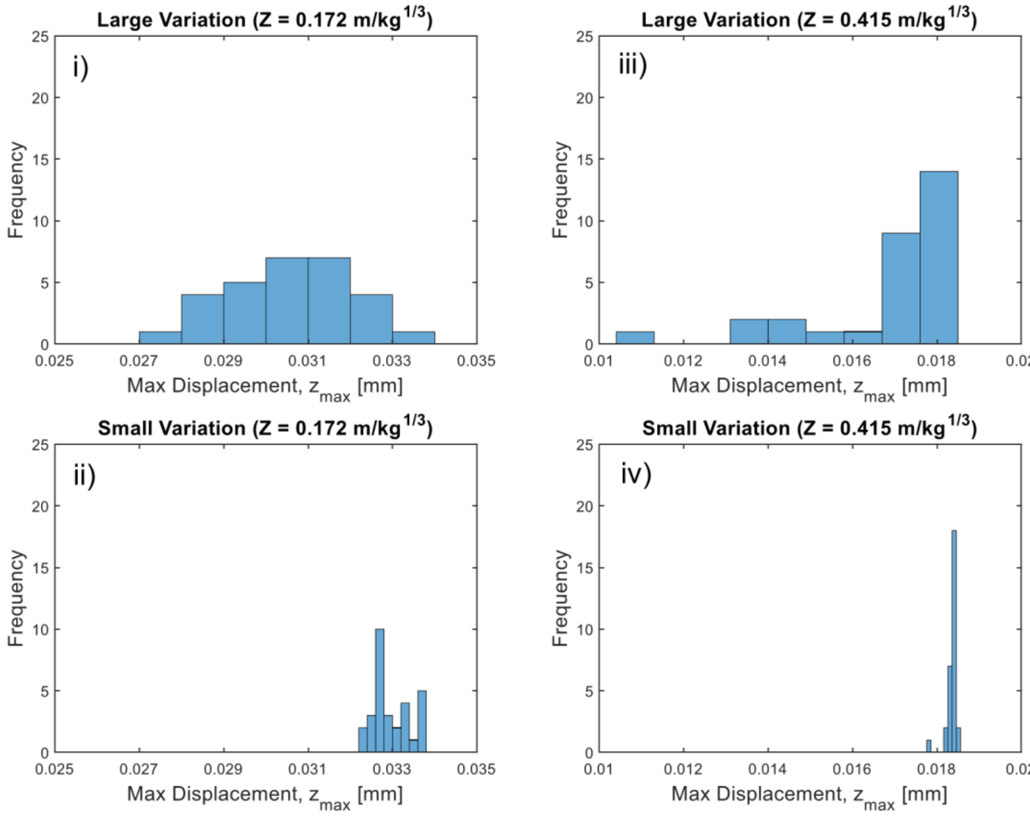

Figure 3: Histograms of z_{max} from Monte Carlo Analysis Set A.

Ref	Variation	Scaled Distance, Z [m/kg$^{1/3}$]	Output standard deviation, σ_z [mm]	Output Mean, μ_z [mm]
i.	Large Variation	0.415	1.805	16.844
ii.	Small Variation	0.415	0.123	18.348
iii.	Large Variation	0.172	3.060	30.027
iv.	Small Variation	0.172	0.458	32.906

Table 3. Results from Monte Carlo Analysis Set A

Figure 3 displays histograms of maximum displacement (z_{max}) of each model for the four Monte Carlo analyses performed in Set A. As previously discussed, the only source of input variation for Set A is from offset of the specific impulse localisation centre. The mean and standard deviation values of each output sample are shown in Table 3.

By qualitatively comparing the histograms at the two scaled distances, it seems that when $Z=0.415\ m/kg^{1/3}$ there appears to be a skew towards larger values of z_{max}, meaning smaller values are more variable. This infers that as the offset of the localisation centre moves towards the restrained edges of the plate, which is when smaller values of displacement will be achieved, then z_{max} is more variable. It should be noted that to determine the output distribution shape more accurately, more models should be run. However, these analyses serve to give a general description of skew or variability in model outputs.

It is clear from the values in Table 3 that z_{max} was more varied at the smaller scaled distance (0.172 m/kg$^{1/3}$) when the input variation is the same (comparing *iii* and *iv*). This is likely due to the magnitude of specific impulse being globally greater at a smaller scaled distance and the degree of spatial localisation also being increased. This result would be true if the probability/likelihood of variation in loading was the same at both scaled distances. However, the findings of various studies that instabilities increase as the DPC grows (i.e. as scaled distance increases) (Fuller *et al.*, 2016; Tyas, 2019; S E Rigby *et al.*, 2020) and that specific impulse distributions are more consistent and subject to less variability in the extreme near-field (Rigby, Tyas, *et al.*, 2019; Pannell *et al.*, 2021) show this not be the case. This means a more physically valid comparison would be between the results of the Small input variation at $Z=0.172\ m/kg^{1/3}$ and the large input variation at $Z=0.415\ m/kg^{1/3}$ (*i* against *iv*). In this case a significantly greater standard deviation of z_{max} is seen at the larger scaled distance. Although the values of input variation were selected with little physical evidence they serve as an indication of how input variability, which is affected by scaled distance, relates to output variability.

Analysis Set B

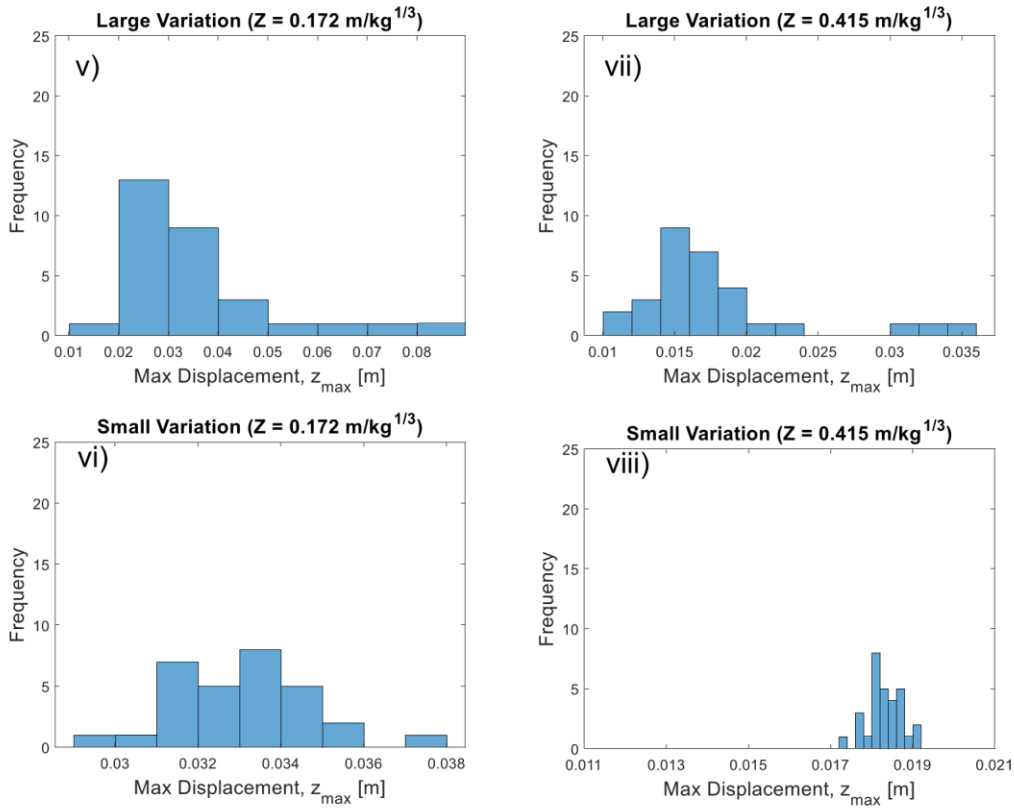

Figure 4: Histograms of z_{max} from Monte Carlo Analysis Set A.

Ref.	Variation	Scaled Distance, Z [m/kg$^{1/3}$]	Output standard deviation, σ_z [mm]	Output Mean, μ_z [mm]
v.	Large Variation	0.415	5.898	17.577
vi.	Small Variation	0.415	0.439	18.268

vii.	Large Variation	0.172	16.230	35.441
viii.	Small Variation	0.172	1.581	33.043

Table 4. Results from Monte Carlo Analysis Set B

Figure 4 shows the histograms of the z_{max} output distribution for Monte Carlo analysis Set B. In these models the scaled distance input value was varied, as well as the offset of localisation centre. This is indicative of the plate being subjected to a protrusion within the DPC or charge shape directionality causing a greater or smaller impulse at the plates location (e.g. a cylindrical charge may cause a larger impulse on the plate from its axial blast wave than a spherical charge at the same stand-off distance).

Comparing values in Table 4 to their respective results in Set A, the standard deviation of z_{max} is significantly increased: *3.3 - 3.6* times greater at $Z=0.415\ m/kg^{1/3}$ and *3.5 - 5.3* times greater at $Z=0.172\ m/kg^{1/3}$. This suggests that either variation in specific impulse is more significant in its effect on z_{max} or its combined effect with offset variation results in a far more significant spread in output results. Similarly to Set A variation of z_{max} is far more significant at the smaller scaled distance when input variation is consistent, which does not account for the interaction between scaled distance and loading variability seen in other studies.

Inspection of histograms *v* and *vii* shows that large input variations result in a skew of the output distribution probability to smaller values of z_{max}. This suggests that models with larger plate deformations are more variable in their maximum response. This can be explained by peak specific impulse increasing exponentially as scaled distance decreases (Pannell *et al.*, 2021), meaning small increases of input scaled distance results in increasingly larger magnitude loadings and therefore greater changes in maximum displacement.

Comparing Figures 4vii and 3iii demonstrates how the skew of the output distribution has been reversed completely when considering a large input variation at $Z=0.415m/kg^{1/3}$.This again suggests that either variation of input scaled distance is more dominant in its influence on displacement than offset of the localisation centre, or that the combined effects of both completely alters the output distribution of z_{max}.

Probabilistic Approach to Fast-Running Models

To demonstrate how a probabilistic approach can be used to quickly establish structural response, the distributions shown in histograms i-viii were individually fit to either a Normal or Weibull distribution depending on the skew of the data. The cumulative probability density function was then plotted for each and are shown in Figures 5 and 6. Using these plots, a blast engineer can determine the likelihood any z_{max} value is not exceeded for a specified variability of loading.

To more accurately model the distributions generated by each Monte Carlo Analysis, the number of sample models should be increased. However, the use of thirty models was significant enough to estimate mean, skew and variance of each output distribution and therefore serves to demonstrate how probabilistic methods can be utilized to define structural response to near-field blast loading.

The most significant limitation of Figures 5 and 6 is that they can only estimate displacement probability for the input variations and scaled distances specified within this study. Furthermore, the variability of the input loading distributions is not physically or experimentally informed. However, it's been shown that formation and development of instability structures and protrusions within the DPC is affected by scaled distance (Tyas, 2019; S E Rigby *et al.*, 2020). This means that if the relationship between the inherent loading variability caused by instabilities in the DPC and scaled distance can be defined then probability plots such as Figures 5 and 6 can be defined for a range of scaled distances. In this way a means of rapidly assessing the likelihood of a given structural response can be achieved.

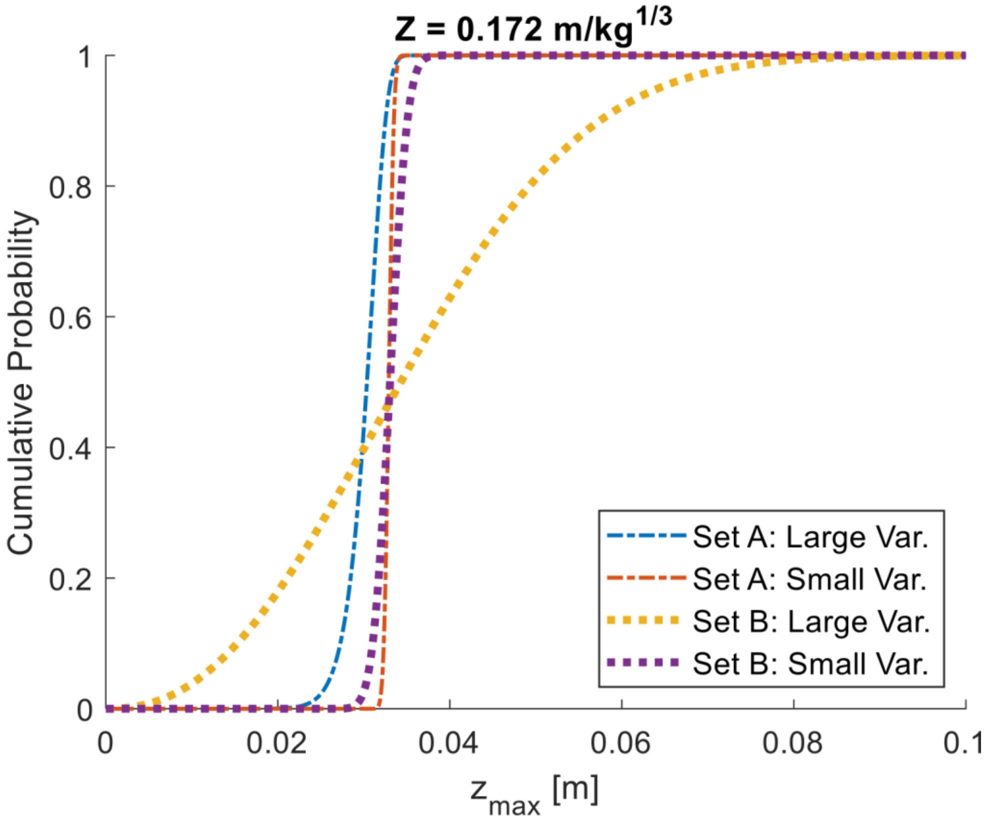

Figure 5: Plots of Cumulative probability distribution for continuous distributions fitted to the results Monte Carlo analyses performed at a scaled distance of 0.172 m/kg$^{1/3}$ in Set A and B.

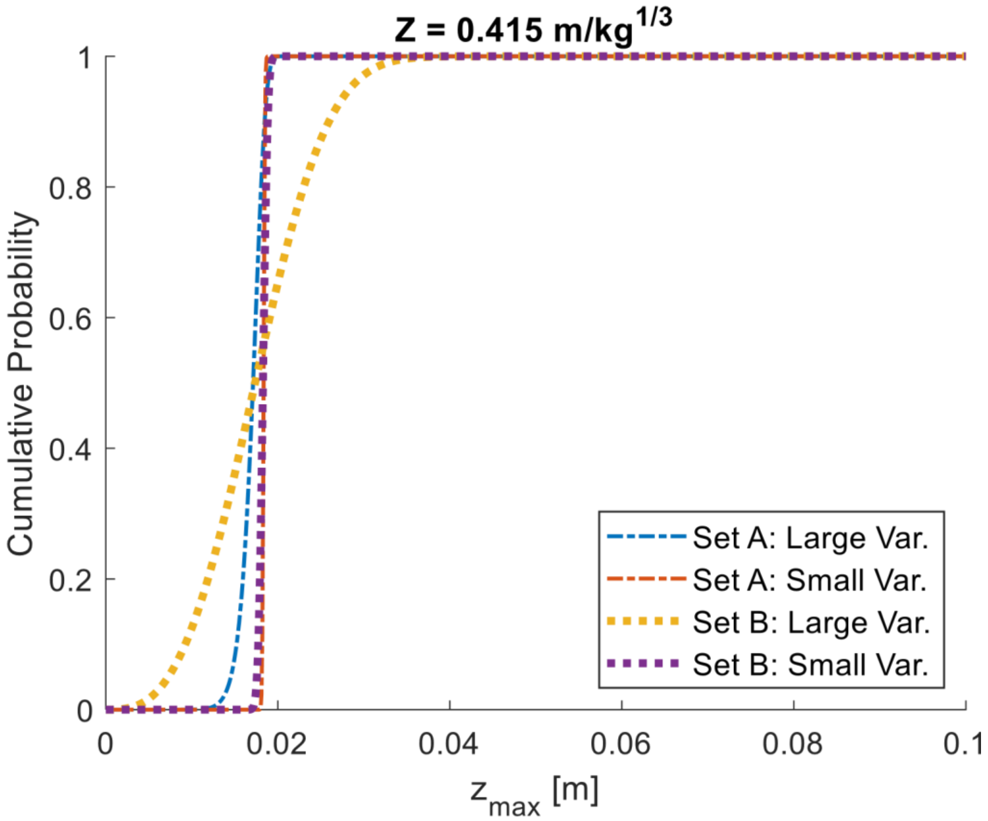

Figure 6: Plots of Cumulative probability distribution for continuous distributions fitted to the results Monte Carlo analyses performed at a scaled distance of 0.415 m/kg$^{1/3}$ in Set A and B.

Summary and Conclusions

This study aimed to demonstrate how probabilistic methods can be used to tackle inherent variability in near-field blast loading and help determine structural response. This was achieved using a family of Monte Carlo analyses consisting of 240 models. The key findings and points of this study can be summarized as follows:

- Variation in blast loading specific impulse distributions results in a variable plate response. • Variance in scaled distance parameters combined with changes in loading localization offset causes a far greater degree of maximum plate displacement variability than if scaled distance variability is ignored.
- If variance of loading parameters is considered constant at all scaled distances, then maximum displacement becomes more changeable at smaller scaled distances if variability of loading is not modelled against scaled distance in a physically informed manner.
- Probabilistic methods can be used to rapidly identify the probability of a given displacement of a structure being achieved if the loading variability is known.
- Interaction between scaled distance and inherent loading variability needs to be defined if near-field blast loading and plate response are to be modelled using fast-running methods.

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